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Original scientific paper

AN OPTIMIZATION OF OUTPATIENTS' WAITING TIME AND HEALTH-RELATED RISKS

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Abstract

The study of optimal queuing systems in healthcare is crucial at such a time as this to help decongest the system, and minimize financial losses and health-related risks associated with long waiting queues. This study examined a queuing system at an outpatient hospital clinic post intending to minimize waiting time in association with financial cost and healthcare-related risks. We observed the queuing system using the sampling survey information of 200 outpatients that visited the clinic for 4 weeks. We used the initial queuing ground truth parameters as the baseline scenario and further simulated 4 other queuing scenarios using the TORA optimization software. We calculated the total expected cost associated with the server(s) (Doctors) and the patients while in the queuing system for each scenario. We further discretize their health-related complications and calculated the incidence rate of the patients while in the queuing system to evaluate their health-related risks. The findings of our study showed there is an association between patient waiting time, financial cost, and health-related risks. Also, the proposed queuing model showed that the system utilization, optimal expected total cost, health-related risks (risk of discomfort and illness/infections developed while in the queue), and waiting time are optimal at the hospital clinic with 5 servers (doctors). The major contribution of this study arose from the incorporation of financial costs and health-related risk variables into the proposed patient queuing model to minimize patient waiting time at the outpatient clinic.

Keywords: Outpatient, Queuing system, Patient waiting time, Financial cost, Health-related risk

1. Introduction

Healthcare emergencies are rapidly on an increase, especially in the wake of the COVID-19 pandemic and other health-related risk issues. Lengthy outpatient waiting time resulting in health-related risk is becoming a thing of concern in modern times. The healthcare system in recent times has been plunged into a crisis due to limited resources and infrastructure to cater to healthcare emergencies. The outpatient clinic is a critical aspect of the healthcare system that performs crucial roles such as preventive medicine, diagnostic, or treatment services to patients that are not required to stay overnight at the medical healthcare facilities. For example, some minor surgeries are performed as outpatient procedures as are pain management, chemotherapy, wound care, physical therapies, consultations, and more. With the advancement of many medical technologies and techniques, more procedures can be safely performed on an outpatient basis than in the past, this has helped to decongest the healthcare facilities and also reduce healthcare-related costs and risks.

Outpatient clinics have continued to witness an upsurge in the number of patients that seeks different medical attention and treatments. Also with all attention focused on addressing the COVID-19 pandemic crisis and other healthcare emergencies, there has been a drastic shortage of healthcare manpower and resources to match this sharp increase in demand for healthcare services even in developed countries. The story is the same or even worst when one considers the plight constantly faced by outpatient clinics in developing countries like Nigeria. Patients who cannot afford private healthcare centers and opted for public clinics usually spend hours if not days in lengthy queues trying to see a consultant or get treatments.

In Nigeria, one of the most populous countries in Africa, the healthcare crisis at the outpatient clinic due to lengthy queues is completely an eye-saw that has become a source of concern, especially in the wake of the coronavirus pandemic [20]. Several outpatients visit healthcare centers or hospitals daily to receive various forms of medical aid and consultation services. These patients usually spend several hours in queues before being attended to by healthcare providers [11]. Some patients have unfortunately lost their lives while some have suffered various forms of complications due to this delay. This has resulted in many complaints and some cases lawsuits against the hospitals. As the world continues to battle with the coronavirus, it has become pertinent to reduce overcrowding in public places especially hospitals to reduce the spread of the virus [7].

There are unique challenges in working in healthcare institutions across Nigeria today even as the country struggles to balance its increasing population. Most Doctors for example manage a lot of medical complications, but at the cost of long working hours demanding physical and mental stress. There are several reports of patients collapsing while in queues, discomfort, and feeling fatigue may lead to health-related complications [6]. Lack of adequate space compromises both patients' and doctors' privacy and increases the risk of cross-contamination [19]. Moreover, increased patient flow wastes most of the time available to a server (doctor) in completing paperwork. Inciting violence against doctors may be attributed to long waiting times and insufficient resources for lifesaving interventions. Therefore, more research is necessary to identify the best queuing model scenario that minimizes cost and health-related risks in the context of an outpatient clinic in Nigeria. To further address the present gap in knowledge and proffer solutions that could augment the current situation at the outpatient clinic queuing post, we set up the following hypotheses. 1) Patients and the hospital management do not usually lose resources in a long queuing scenario at the outpatient clinic department; 2) Health medical experts (servers) and outpatients (customers) are satisfied with the current queuing situation (waiting and service time) at the outpatient clinic department; 3) Patients are exposed to various form of health-related risks while in the queuing system at the outpatient clinic department of the hospital. All the hypotheses were stated in their null form. To validate the stated hypotheses, this study aimed to develop a queuing model to minimize outpatients' waiting time, financial costs, and health-related risks while in the queuing system at the outpatient clinic. The specific objectives of this study: 1) to explore using descriptive statistics the best variables that can be used to model the outpatient clinic queuing system; 2) to optimize the queuing system thereby identifying the best queuing scenario that minimizes patient' waiting time, financial cost, and health-related risk; 3) to establish a relationship between patients waiting time, financial cost, and health-related risk.

We proposed a robust multi-server (M/M/C/FCFS/∞/∞/) queuing system that incorporates the cost and a health-related risk model into the queuing model. Queuing models are often applied in service system administration especially when there is uncertainty in the arrival time and service time [4]. Queuing models unlike other methodologies are easier and cheaper to develop because they require little data and they yield simpler results for forecasting performance measures such as expected delay, and the probability of waiting longer than a given time before being serviced [7].

The rest of this study is organized as follows; in section 2 we discussed the Related works, section 3 focused on the methodology used by this study, in section 4 we presented the results of the findings, in section 5 we discussed the results, and in section 6 we concluded the study and made further recommendations.

2. Related works

A study conducted by Oladejo, & Aligwo [16] examined the queuing system at a medical center to proffer alternative solutions to a congested queuing system. A single queue and three servers were used in the out-patients department of the hospital and the analysis carried out showed that the baseline service intensity of the queue was 80%. They further improved this result by considering an additional number of doctors in the department from 3 to 4. This in turn showed tremendous results as the fatality rate at the hospital decreased drastically to half of its initial state.

During the first wave of the coronavirus pandemic, several scientific studies were conducted to proffer recommendations for queue management to facilitate operational efficiency in the outpatient department and to also speed up service delivery during a future pandemic [13]. Time measurements were observed for each patient arriving at the healthcare point, this has served as a selling point in understanding the current existing queuing problem in many healthcare units and proper actions and solutions were preferred [11], [21]. The study of Nawusu, & Danaa et al., [13] for example analyzed the queuing situation of folder collection at a health medical center. Their study recommended computerized folder system be adopted for tracking, and a more spacious waiting space should be made available for patients so that social distancing can be maintained. Having a functioning smart health care system with less waiting time in queues has no doubt minimized health emergencies and the risk of dying patients in developed countries [2]. There have been reports of decreased human servers' physical or mental well-being which has become a global concern [20], especially in the surge of the COVID-19 pandemic. Chronic stress or illness at work has been linked to an increased risk of cardiovascular diseases, decreased immunity, and depression [9]. Most low/middle-income countries today are struggling with their healthcare infrastructure and a massive exodus of healthcare personnel to advanced nations for well-paid jobs [14]. This leads to overcrowding especially at state-owned hospitals causing burnout, decreased mental health, increased working hours, and sleep deprivation [21]. According to Day [5], one of the biggest factors in the rising costs of health care is chronic illness. Delayed care often transforms an acute and potentially reversible illness or injury into a chronic, irreversible condition that involves permanent disability. Day [5] further states that scientific literature is increasingly reporting harm related to long wait times, including poorer medical outcomes from care and an increased risk of adverse events. While the review of studies below is by no means exhaustive, it reveals that rationing health care by waiting is extremely costly.

3. Material and method

This study adopted several existing approaches as reviewed in the literature, however, the unique contribution of this study arises from the introduction of a risk model to the proposed queuing model of Onoja, & Kembe [18]. The following methodological steps were utilized by this study in achieving the results: 1) Sampling techniques and data collection procedure, 2) queuing model parameters, 3) introducing financial costs into the queuing model, 4) introducing health risk into the queuing model, 5) analysis with TORA optimization software.

3.1. Sampling Techniques and Data Collection Procedure: This study utilized the simple random sampling approach to randomly draw a sample of 200 patients from an unlimited (N) population of patients that arrive at the outpatient unit of the hospital clinic for four weeks. A descriptive sampling survey questionnaire was used to collect the patients' consent information using the electronic survey questionnaire tool called ODK. The sample-sized calculation was done using:

$$sample\ size\ (n) = \frac{Z^2 \times P(1-P)}{e^2} \tag{1}$$

Where $N =$ population size, $e =$ margin of error, $Z =$ Z-score (the number of standard deviations given the population is away from the mean), this study uses the $Z = 1.96$ (95%), $Cl =$ confidence interval to determine the margin and error. Since our sample size is drawn from an unlimited population size, we defined the proportion of the population as $P = 0.5$, and the margin error $= 0.05$. Thus, the sample size required was calculated as:

$$sample\ size\ (n) = \frac{(1.96)^2 \times 0.5(1-0.5)}{0.05^2} = 384.16 \tag{2}$$

The raw data from the sampling survey information collected via ODK reported some missing information and others declined consent to participate in the survey. 153 patients' information was successfully captured and cleansed using Excel for further analyses and visualizations.

3.2. Queuing Model Parameters:

Procedures of System Parameters Estimation

The system performance parameters used in the study were defined as follows:

λ = arrival rate of patient per unit time.

λ_{eff} = **λ,** because there is no limit on the number in the system;

μ = service rate (length of stay) of patient per unit time;

C = Number of doctors (servers) in this model, there are C parallel servers.

ρ = Hospital healthcare system utilization factor i.e., the fraction of time servers (Doctors) are busy, this is equals $\frac{\lambda}{(C\mu)}$, since the recent study adopted multiple–server models.

L_q : average number of patients in the queue $= \left\{\frac{P^{c+1}\mu\lambda}{(C-1)!(C-P)^2}\right\} P_0$ (3)

L_s : average number of patients in the system = $L_q + \frac{\lambda}{(C\mu)}$

W_q : Average waiting time of a patient in the queue = $\frac{L_q}{\lambda}$

W_s : Expected waiting time of patient in the system = $\frac{\boldsymbol{L_s}}{\boldsymbol{\lambda}}$

P_0 = possibility of 0 patients existing in the system.

$L_n =$ Expected number of patients waiting in line excluding those times when the line is empty.

$$P_n = \begin{Bmatrix} \frac{\lambda^n}{\mu(\mu)(\mu)\ldots(\mu)} P_0 = \frac{\lambda^n}{n!\mu^n} P_0 = \frac{P^m}{n!P_0}, & n<C \\ \frac{\lambda^n}{\mu(\mu)\ldots((C-1)\mu)(\mu^{n-C+1})} P_0 = \frac{\lambda^n}{C!C^{n-C}/n} P_0, & n \geq C \end{Bmatrix} \tag{4}$$

$$P_0 = \left\{\sum_{n=0}^{C-1} \frac{P^n}{n!} + \frac{P^C}{C!}\left(\frac{1}{1-\frac{P}{C}}\right)\right\}^{-1}, \ \frac{P}{C} < 1 \tag{5}$$

3.3. Introducing Financial Costs into the Queuing Model: To evaluate and determine the optimum number of servers in the system, two opposing costs must be considered in making these decisions: Service costs, and waiting-time costs of patients. Economic analysis of these costs helps the management to make a trade-off between the increased costs of providing better service and the decreased waiting time costs patients derived from providing that service.

The objective is to minimize the total expected system cost

Expected service cost $E(CC) = CC_s$ (6)

Where, C = number of servers, C_s = service cost of each server

Expected waiting costs in the system $E(WC) = (\lambda W_s)C_w$ (7)

Where, λ = number of arrivals,

W_s = average time an arrival spends in the system.
C_w = opportunity cost of waiting by the patients. Adding (1) and (2) become:
Expected Total costs $= min\{E(TC) = E(CC) + E(WC)\}$ (8)
Expected Total costs = $min\{E(TC) = SC_s + (\lambda W_s)C_w\}$ (9)
$E(TC) = C_O + CC_S + (\lambda W_S)C_W$ (10)
C_o : The fixed cost of the operating system per hour
C_s : The marginal cost of a registration agent (doctor) per hour.

According to Onoja & Kembe [18], an analytical solution does not exist for this problem. Thus, it is necessary to solve the problem by grouping to solve the convex function problem**.** Therefore, it just takes to estimate $E(TC)$ for the values of C grouping up until the cost ceases to decrease.

3.4. Cost of Waiting Time (WC)*:* One unit of waiting time by a patient was estimated based on their monthly-earned allowance.

The mean waiting time cost per hour of the patient is:

$$C_W = \sum W_i \, C_i; \tag{11}$$

Where: C_i : Hourly earned income of a patient in a group/category, W_i : Weight of the category i is extracted from the total. To calculate the marginal cost, hourly charges were used for consistency. Crucial assumptions were made: 1) Medical doctors (servers can be experienced consultants or resident doctors) are paid a monthly-earned allowance, a flat rate of # 400,000., 2) Medical doctors (servers) attend to the patients between the time frame of 8 a.m. – 5 p.m. (total of 9 hours in a day); 3) Hospital management claimed to spend on hourly basis # 56,839 on maintenance and servicing of queuing facilities e.g. desktops, cooling systems, sanitation, hospital equipment, etc.; 4) Servers work for 5 working days (Monday – Friday), note: shifting was ignored. 5) 4 weeks in a 12-month calendar for a year were utilized.

3.5. Introducing Health Risk into the Queuing model: Risk in this context refers to the probability that a patient develops a specified illness/disease over a specified interval of time (while in the queuing system at the hospital), given that the patient is alive and illness/disease-free at the start of the period (arrival in queuing system). Calculating the incidence refers to new cases of disease occurring among previously unaffected individuals. The population incidence rate for this study is the number of new cases of the illnesses/diseases occurring in the population of patients in queue throughout their entry into the queuing (inter-arrival time) divided by the sum of observation times (total time spent in the queuing system at the hospital), on all patients who were disease-free at the beginning of the time interval (arrival in the queuing system). Based on this study context, an incidence rate is time-dependent and depends on both the starting point (arrival in the queue at the hospital) and the length of stay of the patient in the queuing system. With the data set obtained from issuing descriptive sample questionnaires to the patients on arrival at the hospital queuing system, this study estimates the incidence rates by partitioning the patients' arrival time (minutes) λ_j at the queuing system into intervals of lengths L_j having midpoints (average time) t_j for $j = 1, ... J$, and estimating a rate for each interval. Let n_j denote the number of patients who are illness/disease-free while in the waiting line at a time t_j in the queuing system at the hospital, and d_j the number of new complaints of illness/diseases reported having suffered from during the jth interval in the queue. An estimate of the incidence rate at the time t_j will become:

$$\hat{\lambda}(t_j) = \frac{d_j}{n_j L_j} \tag{12}$$

The denominator in $\hat{\lambda}(t_j)$ approximates the sum of observation times on the n_j population members in the jth interval. In practice, this is usually replaced by the actual observation time the patients spent in the queuing system. This implies that the d_j diagnoses of illness/disease did not occur exactly at the time t_j risk.

3.6. Introducing the Queuing Health Risk Model: The queuing health risk model in the context of this research is defined as the probability that a patient at the hospital develops a specified symptom(s), illness/disease during their stay or waiting in the entire queuing system at the hospital given that the patient is alive with no symptom illness/disease developed on arrival in the queuing system. As the incidence rate, the risk is time-dependent and depends on both the starting point and the length of stay of the patient in a queuing facility. The proportion of new occurrences d_j among n_j disease-free patients in the queue at the time t_j:

$$\hat{P}(t_j) = \frac{d_j}{n_j} \tag{13}$$

is an estimate of the risk or probability of a patient developing a symptom(s), illness/disease occurrence in the jth time waiting time in the queuing system.

The incidence rates and risks are related via the general formula:

$$Risk = rate\ symptom(s), \quad disease \times waiting\ time\ in\ the\ queue \tag{14}$$

Mathematically this can be expressed as:

$$\hat{P}(t_j) = \hat{\lambda}(t_j)L_j \tag{15}$$

Combining equation (10) and (15);

$$E(TCR) = C_O + CC_S + (\lambda W_S)C_W + \hat{\lambda}(t_j)L_j \tag{16}$$

Where $E(TCR) =$ the expected total cost with the health risk of a patient waiting in the queuing system.

3.7. Analysis with TORA optimization software: This is a windows-based software that is widely used to automate operational models such as linear programming, transportation models, queuing models, project planning, etc. With regards to the TORA optimization software, we input the average waiting time, average arrival rate, and service rate parameters calculated from the survey questionnaire dataset we collected for each of the patients. Since we seek to optimize the queuing model by minimizing the financial cost and health risk of the patients and the servers, we simulated five scenarios and selected the scenario that best optimized the system.

4. Results

To achieve the aim and objectives of this study, we presented the results of our analysis in chronological order. First, we visualized the research descriptive questionnaire response from the target samples to understand relevant patterns and clues that will be used for the subsequent analysis. Secondly, we used the patients' feedback (waiting time, service time, financial cost-related variables, and health-related risk variables) to minimize the queuing system and identified the best queuing scenario. Lastly, we employed the use of Pearson Correlation coefficients to examine the relationship between the waiting time, financial losses, and health relative risk variables.

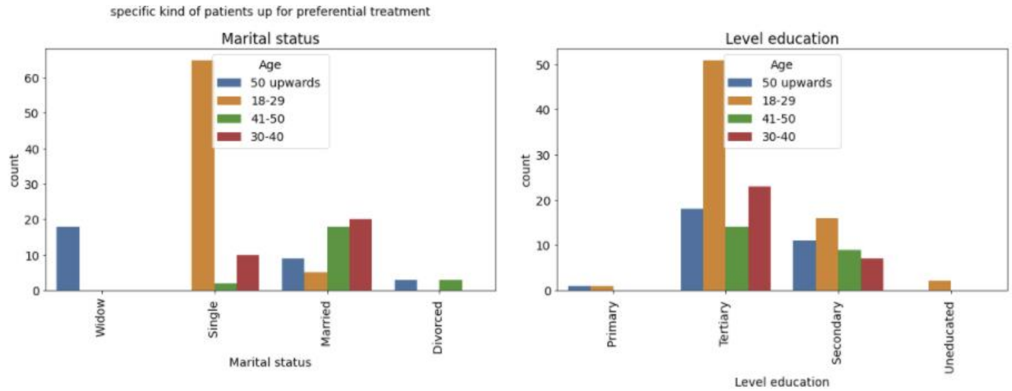


Fig 1. Descriptive Statistics of Demographic responses from sampling survey of the patients

Most of the respondents (see **Fig** 1) that sought medical attention at the outpatient clinic were single persons within the age range of 18 – 29, they mostly attend tertiary institutions. Surprisingly, widows and divorced respondents who are within the age range of 41 – upward has a low turnout for outpatient healthcare services. Also, primary school and uneducated persons were very few and may likely be in the categories of the widow and divorced that rarely seek medical attention.

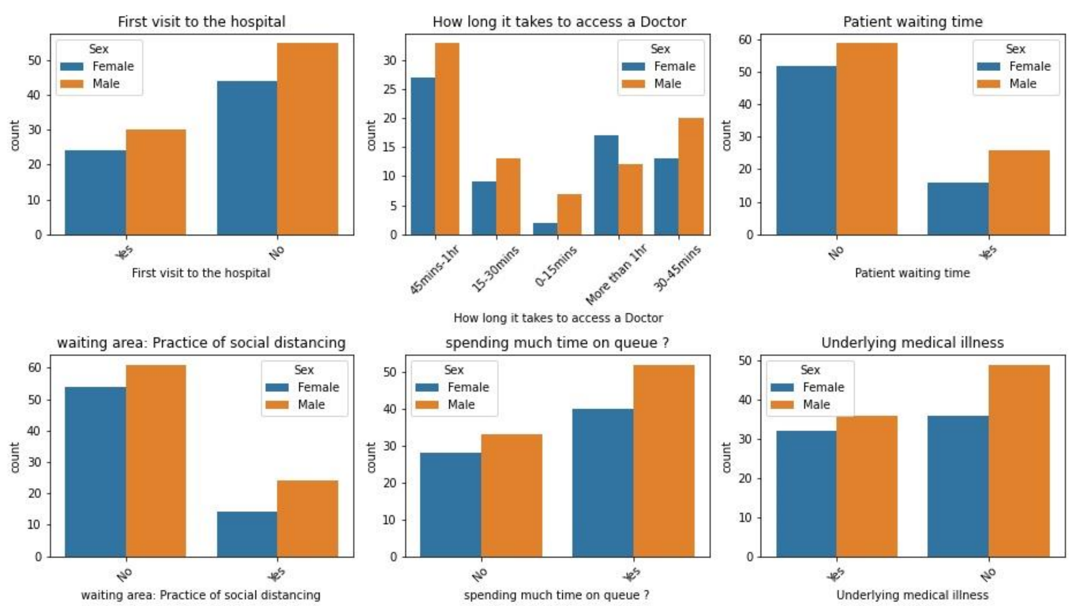


Fig 2. Patients' sources of concern based on gender during a long waiting period at the hospital

The majority of the patients (65%) said this is not the first time they visit the outpatient clinic while 35% of the respondents said this is their first time (this signifies an increase). This statement validates our earlier assertion that there is an upsurge in the number of patients that seeks medical attention at the outpatient clinic in Nigeria. On average, the patients spent 30 minutes waiting in the queue before being attended to by medical experts. Though about 39.2% of the patients spent 45minutes – 1 hour, 21.6% spent 30-45minutes while 19% of the patients spent more than 1 hour waiting. More so, 73% of the patients agreed that the waiting time in the queue is OK, 27% of them however are not in support of this notion. Since this study was conducted during the COVID-19 pandemic, we saw the need to assess the patients' practice of social distancing while in the queueing facilities. Approximately 75.2% of the patients said they do not comply with the social distancing rule, and only 24.8% of the patients observed social distancing rules while in the queuing facilities. About 60.1% of the patients agreed that they spent a lot of time in the queuing system while 39.9% disagreed with this notion. Approximately 55.6% of the patients said they do not have any underlying medical conditions while 44.4% of the patients said they have underlying medical conditions.

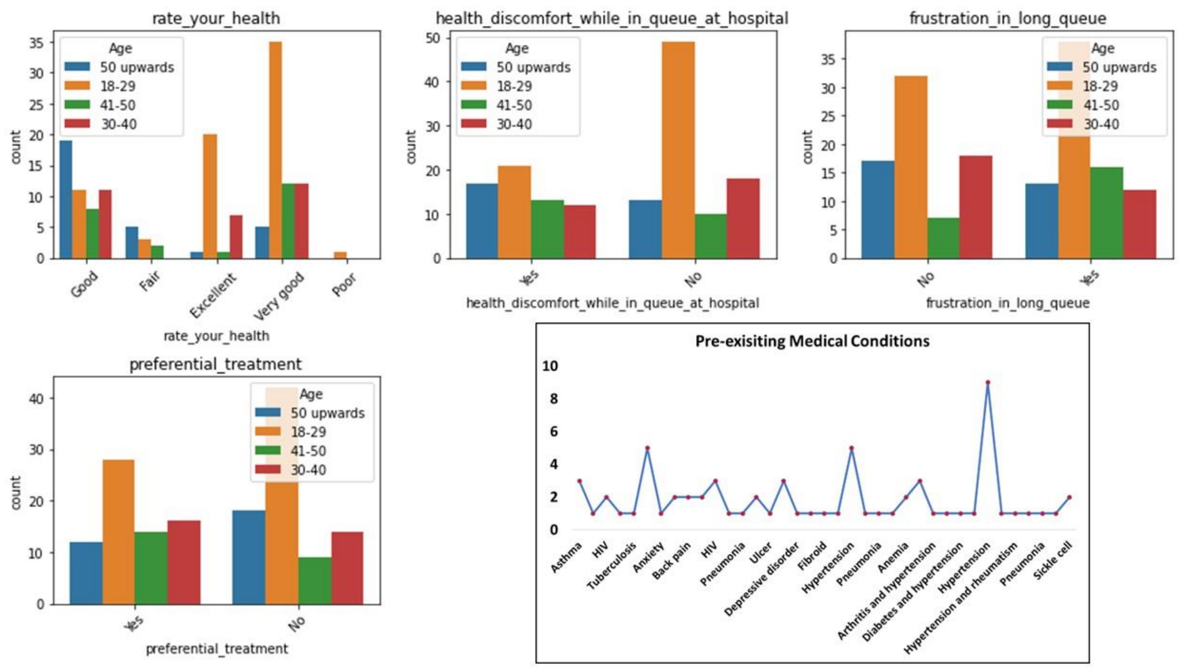


Fig 3. Patients' health-related risk responses based on their age (years) while in a long waiting queue

The majority of the patients who rate their health status as very good (92.8%) were young persons within the age range of 18 – 29 years, and less than 1% of the patients rated their health status as poor. About 58.8% of the patients said they do not feel discomfort while in the queuing system while 41.2% of them felt discomfort. We further investigated patients' pre-existing medical conditions to assess their potential risk. Interestingly about 20.6% of the patients whose age ranges from 41 years and above reported to be suffering from hypertension, other pre-existing medical illnesses common to this age grade category include HIV with about 4.4% and Arthritis with about 4.4%. while about 7.4% of young patients between the ages of 18 – 29 years reported suffering from an ulcer, and another 4.4% suffered from asthma.

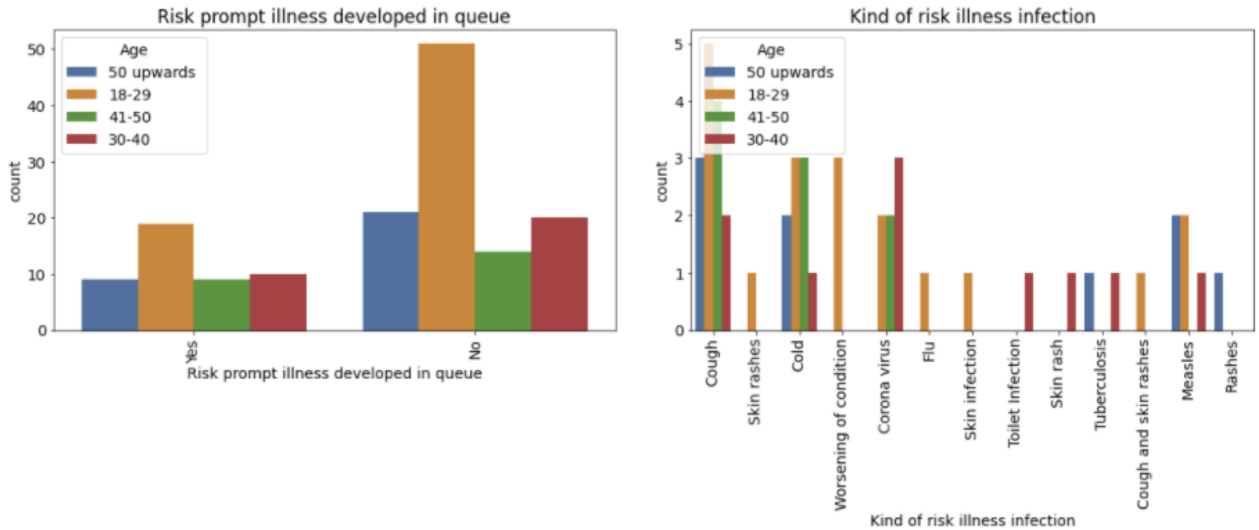


Fig 4. Patients' Health-related risk profile

Majority of the patients 69.3% said they are not prompt to any form of risk of developing illness while in the queuing system while 30.7% of the patients said they are prompt to the risk of developing some form of illness while in the queuing system. Moreso, the highest category of patients that said yes (12.4%) is coming from patients within the age range of 18 – 29, followed by 30 - 40 years (6.5%). Approximately 19.2% of the patients in all the age categories reported being at risk of contracting a cold while in the queuing system, and only 2.2% of the patients in the 18 -29 age reported the risk of flu and also skin rashes, 14.9% in all the age groups reported the risk of contracting COVID-19 infection while in the queuing system, 19.1% of the patients in all the age groups reported the risk of skin rash infections.

To establish the relationship between waiting time, financial losses, and health-related risk, we performed a correlation analysis.

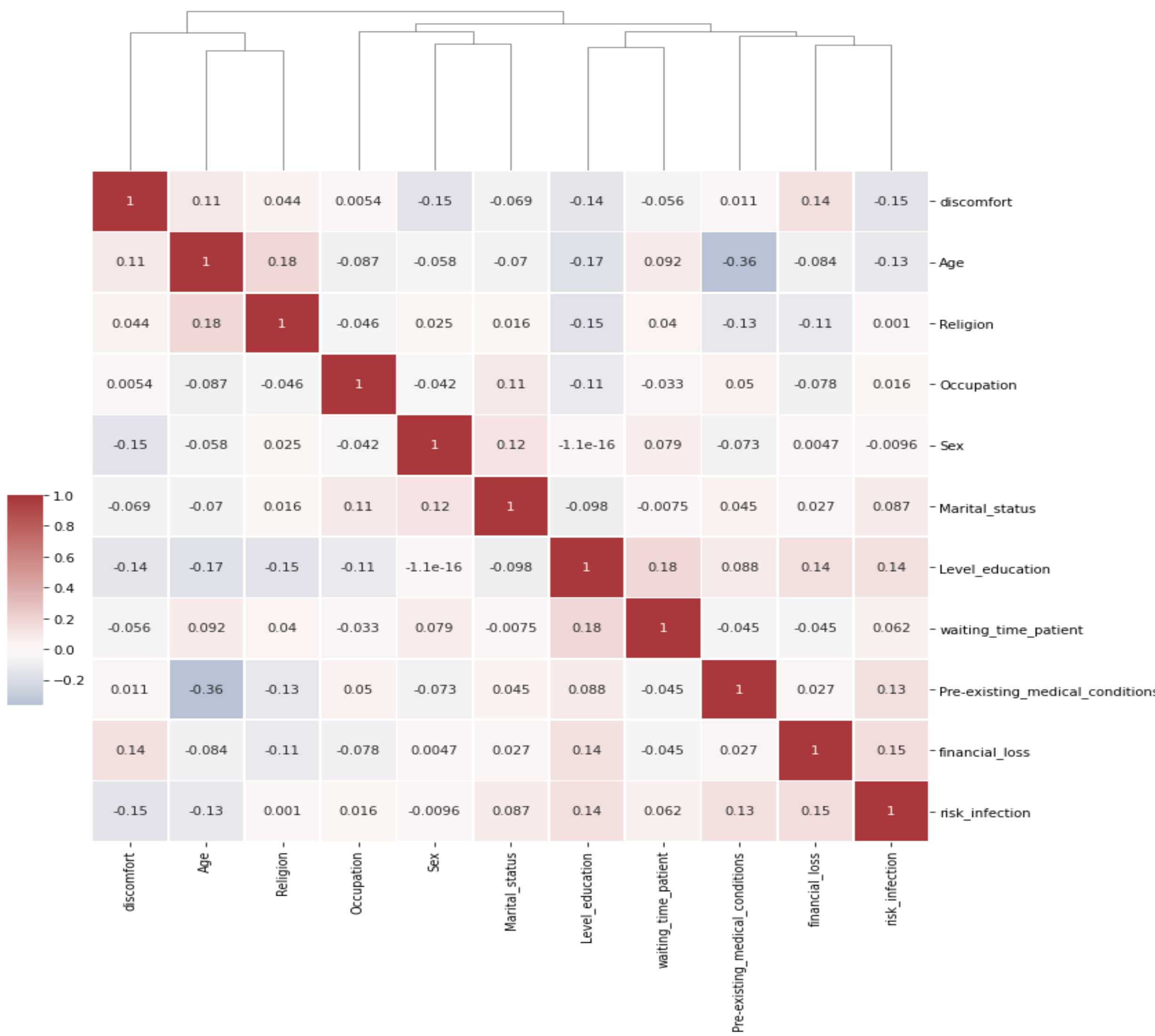


Fig 5. Correlation matrix of Patients' Waiting time, financial losses, and Health-related risks variables.

The patient waiting time (Yes, or No) response in the descriptive questionnaire was binarized and used as the target variable to correlate with the financial and health-related risk variables. Religion and Age, waiting time and educational level have the highest positive correlation coefficient of 0.18, and pre-existing medical condition and age have the strongest negative correlation coefficient of -36. These findings reflect the true state in most African countries where religion, age, and level of education play a crucial role in our daily lives. Queuing is not an exception, for example, the Muslim community prays 5-times a day, and a long queue even in the outpatient clinic can affect a Muslim patient's daily prayer routine.

4.1. Queuing Modeling Analysis Results: Here we presented the results of our findings from modeling the queuing situation at the outpatient clinic using the proposed M/M/C/FCFS/∞/∞/ queuing model.

Table 1. Input parameters for TORA optimization window-based ® software

Parameter M/M/C: FCFS/∞/∞	Value
Arrival rate(λ)	7 patients per hour
Service rate(μ)	2 patients per hour
Number of servers	4, 5, 6, 7, and 8 depending on the scenario

Table 2. Performance measures of M/M/C/FCFS/∞/∞

Scenario	**C**	**Lambda**	**Mu**	**L'da ef**	p_0	L_s	L_q	W_s	W_q
1	4	7	2	7	0.01475	8.66503	5.16503	1.23786	0.73786
2	5	7	2	7	0.02590	4.38162	0.88162	0.62595	0.12595
3	6	7	2	7	0.02896	3.74845	0.24845	0.53549	0.03549
4	7	7	2	7	0.02984	3.57620	0.0762	0.51089	0.01089
5	8	7	2	7	0.03010	3.52324	0.02324	0.50332	0.00332

Table 3. Queuing system and Cost Analysis All grouping

Scenario	**(C)**	**Λ**	**Ws**	**ΛWs**	**Co (#)**	**Cs (#)**	**Cw (#)**	**CCs (#)**	**(λWs)Cw (#)**	**E(TC) (#)**
1	4	7	1.23786	8.66502	56839	238.0952	372000	952.381	3223387	3281179
2	5	7	0.62595	4.38165	56839	238.0952	372000	1190.476	1629974	1688003
3	6	7	0.53549	3.74843	56839	238.0952	372000	1428.571	1394416	1452684
4	7	7	0.51089	3.57623	56839	238.0952	372000	1666.667	1330358	1388863
5	8	7	0.50332	3.52324	56839	238.0952	372000	1904.762	1310645	1369389

Note: this calculation was done for all the 153 patients in all groups while in the queuing system

Table 4. Calculating for Incidence and Risk by patients while in the queuing system

Discomfort felt at the hospital	**No. patients with discomfort (d1)**	**Incidence rate1**	**Risk discomfort**	**Infection contracted**	**No. patients with infection (d2)**	**Incidence rate2**	**Risk of illness/infection contracted**
Tiredness	28	0.061	0.384	Cough	16	0.035	0.022
Worsening of condition	7	0.015	0.009	Cold	10	0.021	0.013
Drowsiness	7	0.015	0.009	COVID-19	7	0.015	0.009
Headache	6	0.013	0.008	Measles	5	0.011	0.007
Pain	11	0.024	0.015	Worsening of condition	3	0.006	0.004
Breathing difficulty	3	0.006	0.004	Tuberculosis	2	0.004	0.003
Nausea	1	0.002	0.001	Skin infection/Rashes	5	0.011	0.007

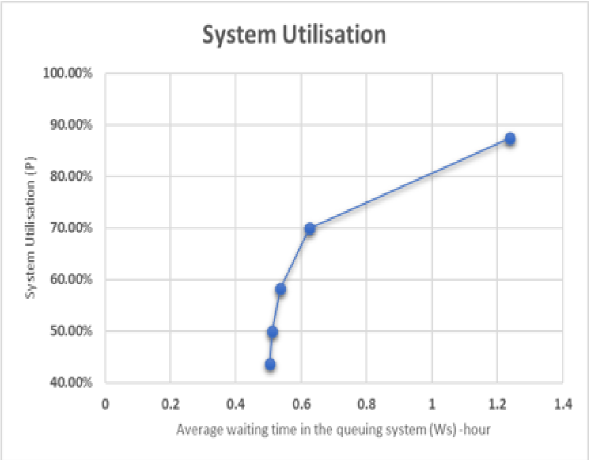
Low Blood pressure	1	0.002	0.001	Toilet infection	1	0.002	0.001
Irritation	1	0.002	0.001	Nausea	1	0.002	0.001

There were 153 sampled patients used for this study. ***Note:*** *for this study, the average time a patient spent in the queuing system at the hospital was utilized = 37.56 minutes (0.626 hours). The overall incidence rate of discomfort felt by patients while in the queuing system at the hospital = 0.1372 = 14%, Overall risk of patients experiencing discomfort felt by patients while in the queuing system at the hospital = 0.1372*0.626 = 9%. The overall incidence rate of illness/disease developed by patients while in the queuing system at the hospital; is 0.1055 = 11%, Overall Risk of patients experiencing or developing illness/disease while in the queuing system at the hospital; is 0.1055*0.626 = 7%.*

Table 5. – Summary of the Marginal Costs, Service Costs, Opportunity, Health-related Risks Cost, and the Number of Servers across the 5 scenarios.

Performance Measures	**Scenario 1: 4 Doctors**	**Scenario 2: 5 Doctors**	**Scenario 3: 6 Doctors**	**Scenario 4: 7 Doctors**	**Scenario 5: 8 Doctors**
Arrival rate (λ)	7	7	7	7	7
Service rate (μ)	2	2	2	2	2
System Utilization	87.50 %	70.00 %	58.33 %	50.00 %	43.75 %
L_S	8.66503	4.38162	3.74845	3.5762	3.52324
L_Q	5.16503	0.88162	0.24845	0.0762	0.02324
Ws - hour	1.23786	0.62595	0.53549	0.51089	0.50332
Wq - hour	0.73786	0.12595	0.03549	0.01089	0.00332
Risk discomfort – hour	10.12 %	1.73 %	0.49 %	0.15 %	0.05 %
Risk illness/infection – hour	7.78 %	1.33 %	0.37 %	0.11 %	0.04 %
P_0	0.01475	0.0259	0.02896	0.02984	0.0301
Total System Cost/hour	#3281178.8210	#1688003.2762	#1452683.5314	#1388863.2267	#1369389.0419

In table 5, scenario 2 (highlighted) was identified as the best scenario that optimizes the patient waiting time, financial losses, and relative health risks.



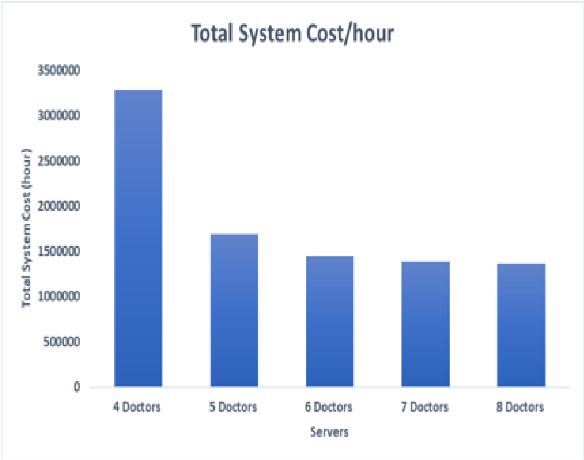


Fig 5. Queuing system sever utilization and total system cost of each server per hour

The line plot (see Fig ure 5) showed a sharp decrease in the patients' waiting time from 4 doctors to 5 doctors and a steep decrease in the rest scenarios. Also as the server decreases the utilization rate increases (servers become busier). The best scenario indicated that the server will be 30% of the time idle, this time relapse can be used for paperwork or entry of patients' information into electronic health record systems.

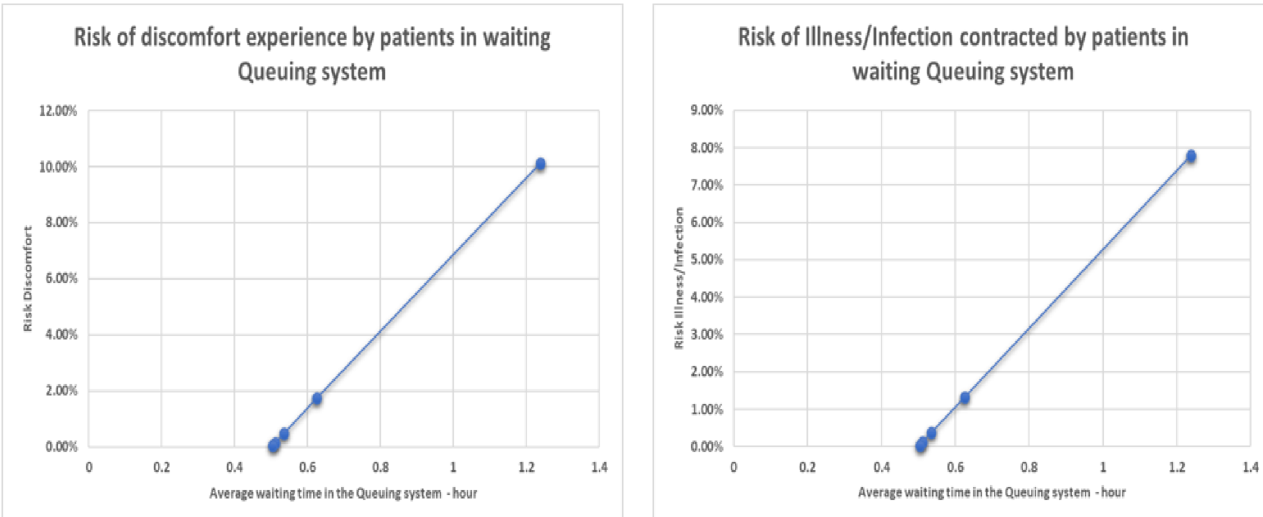


Fig 6. Health-related risk of discomfort associated with a patient while waiting in queue to see the Doctor, and the risk of illness/infection contracted by the patient while waiting in the queuing system.

5. Discussion

In the wake and aftermath of the COVID-19 pandemic, healthcare facilities across the globe have continued to experience a rapid explosion in patients eager to receive treatments or consultation services. There is a huge problem of limited resources such as medical professionals and healthcare facilities to cater to the needs of these patients. Patients' financial losses and health-related risks are crucial factors that have been neglected for a long time by experts while they stay in a long queuing system. The trade-off between the opportunity cost of staying in the queuing system to receive medical attention or balking out of the system to attend to prior needs such as business, faith-related issues, or suffering the risk of contracting disease and infections becomes crucial in modern times. This study, therefore, examines a queuing system at the outpatient clinic of a public hospital in Nigeria to optimize the patient waiting time, financial losses, and health-related issues and to further establish a relationship between patient waiting time, financial losses, and health-related risks. The majority of the patients that were issued the sampling survey questionnaire were in their prime (18 -29 years) and students. They may not likely understand the concept of financial losses in this context but are more likely to express their feelings about health-related risks or discomfort felt while in the queuing system. The result of our findings also showed (see **Fig . 1**) that widows and divorced individuals rarely seek medical treatment at the outpatient clinic. The majority of the patients rate their health as very good and were divided on their opinion about whether the queuing situation is frustrating or not. Many of the patients indicated they have suffered some forms of discomfort while in the queuing system such as tiredness, headache, cold, etc. Most of them also indicated they were at one time or the other exposed to a risk of contagious infections such as cold, cough, COVID-19, measles, skin rashes, and toilet diseases while in the queuing system. Interestingly, the correlation matrix (see **Fig . 5**) quantified the strength of relationships that exist between the variables of interest (waiting time, financial loss, and health-related risks). For example, there are positive associations between the level of education and patients' waiting time, religion, and age, and negative associations between age and pre-medical conditions, sex, and discomfort. These findings reflect the true reality on the ground in most African countries where gender, religion, and level of education play crucial roles in our daily routines.

The result of the proposed queuing model as proposed by this study (see **Table 3**) summarized the different servers and scenarios with corresponding arrival time, average waiting time in the system, fixed cost of operation per hour, the marginal cost of a server (doctor) per hour, cost of waiting time, expected service cost, expected waiting cost, and expected total cost in the system for each scenario. The arrival time, the marginal cost of each server per hour, and the waiting cost were constant. Table 3 showed a steady decrease in the waiting time in the system, expected waiting cost, and expected total cost across the scenarios. While a steady increase in the expected service cost across the scenarios. This implies that increasing the number of doctors will yield an increase in service cost in the sense that the hospital management will require more funds to employ more doctors, but it will drastically reduce the expected total cost of the system.

Queues in hospitals are quite common in developing countries, especially Nigeria. Overcrowding as well as long waiting in hospitals have been a means of contracting infections and medical complications. In this regard, from the result presented in table 4, on average, patients spend at least 38 minutes in queues at the hospital, which amounts to a significant rate of discomfort (14%) felt by patients with a 9% risk rate of patients experiencing discomfort while in queues. This implies that there is a significant probability that most patients will experience discomfort while waiting in hospital queues. Furthermore, with an 11% overall incident rate of disease developed by patients while in queues in the hospital, it shows that a significant number of patients will contract one form of disease or the other while waiting in queues in the hospital.

Table 5 shows that there is a higher percentage of time 4 doctors spend actively working and it steadily decreases as the number of doctors increases. This implies that employing more doctors will significantly reduce the workload, fatigue, and mental stress of the doctors, which in turn will result in more efficient service rendering. There is also a drastic decline in the number average of patients in the hospital queues, waiting time, risk of discomfort, infections, and total system cost. However, the probability of not having any patient in the queue increases as the number of doctors attending to patients increases (server idle time). The results presented showed that scenario 2 with 8 doctors is the most appropriate scenario to achieve optimal cost with a total system cost of *#1688003.28* as against scenario 1 with 4 doctors which has a total system cost of *#3281178.82*.

The findings of this study showed that it is not just the server utilization rate that needs to be optimized to increase efficiency but also good to incorporate certain components such as financial constraints and health-related risks in hospital queuing systems to minimize and prioritize certain health complications. For instance, during the first surge of the coronavirus pandemic, the health sector facilities in most countries collapsed. The high arrival rate at hospital facilities overwhelms the healthcare existing servers. In some advanced countries such as the United States and Italy, priority was given to patients with high mortality rates. Having such components incorporated into the queuing system, especially in healthcare can account for such emergencies and forestall possible alternatives that can sure save lives.

6. Conclusions

In this study, we incorporated financial losses and health-related risks into a queuing model at the outpatient clinic of a public hospital in Nigeria to optimize the patient waiting time. The findings from this study established that there are relationships between financial losses, health-related risks, and patient waiting time at the outpatient clinic. It is necessary to incorporate health-related risks into the queuing model to minimize the incidence rate and complications developed by patients in long waiting queues at healthcare facilities. This is important to help prioritize high-risk patients and increase their survival rate. In the future, we seek to further explore the cross-talking of these health-related risks concerning the patients' queuing parameters and machine learning modeling techniques. Such studies could serve as a good criterion to evaluate the allowable time a patient can wait in a queue with prior complications and plausible recommended alternatives can be put in place in case of unforeseen emergencies capable of collapsing the queuing systems.

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