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Professional paper

USING EULER'S METHOD TO APPROACH THE SOLUTION OF A FIRST-ORDER DIFFERENTIAL EQUATION

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Abstract

The objective of studying this paper is the use of Euler's method to approach the solution of a first-order differential equation at the interval between x_0 and x_F. Euler's method was the most fundamental and simplest of procedures used to find approximate numerical solutions of a ordinary first-order differential equation, provided his initial value is known. In Euler's method, we can approximate the curve of the solution by the tangent in each interval (that is, by sequence of short line segment) at steps of h. In general, if we use small step size, the accuracy of approximation increases.

Keywords: step h, approximate value, the Euler's method

1. Introduction

Differential equation is an equation that relates one or more unknown functions and their derivatives. If the largest derivative that appears in the equation is degree one, then it is a common first-degree differential equation.

Euler's method is quite inaccurate, if the length of step h is great value. It gives an increasingly accurate approximation as the integration step is reduced. If the segment is very large, then each segment is divided into N-segments of integration and Euler's formula applies to each of them by one step, i.e., the step of integration h takes less than the step in which the solution is resolved.

2. Euler's Method of Approaching Resolving the First-Order Differential Equation

Consider the problem of initial value $y' = x^3 + y^2, y(1) = -2$.
The idea behind the areas of direction can also be applied to this problem to study the behavior of its solution. For example, at point $(1, -2)$ the solution slope is given with

$$y' = 1^3 + (-2)^2$$
$$y' = 5$$

So, the slope of the tangent right to the solution at that point is also equal to 5. Now we determine $x_0 = 1$ and $y_0 = -2$. Since the solution slope at this point is equal to 5, we can use the linear approximation method to get closer to the point $(1, -2)$.

$$L(x) = y_0 + f'(x_0)(x - x_0)$$

Here $x_0 = 1$, $y_0 = -2$ and $f'(x_0) = 5$, so linear approximation becomes

$$L(x) = -2 + 5(x - 1)$$
$$L(x) = 5x - 7$$

Now we choose a step size. Step size is a small value, usually 0.1 or less, which serves as an increase for x and is represented by the h variable. In our case, let it be $h = 0{,}1$. Increasing x_0 by h gives our value of the next x

$$x_1 = x_0 + h = 1 + 0{,}1 = 1{,}1$$

We can replace x_1 in linear approximation to calculate y_1. We have

$$y_1 = L(x_1) = 5 \cdot 1{,}1 - 7 = -1{,}5$$

Therefore, the approximate y value for the solution when $x = 1{,}1$ is $y = -1{,}5$. Next, we can repeat the process using $x_1 = 1{,}1$ and $y_1 = -1{,}5$ to calculate x_2 and y_2. The new slope is given with:

$$y' = (1{,}1)^3 + (-1{,}5)^2$$
$$y' = 3{,}581$$

First of all,

$$x_2 = x_1 + h = 1{,}1 + 0{,}1 = 1{,}2$$

Use of linear approximation gives

$$L(x) = y_1 + f'(x_1)(x - x_1)$$
$$L(x) = -1{,}5 + 3{,}581 \cdot (x - 1{,}1)$$
$$L(x) = 3{,}581x - 5{,}4391$$

We replace x_2 in linear approximation to calculate y_2

$$y_2 = L(x_2) = 3{,}581 \cdot 1{,}2 - 5{,}4391$$
$$y_2 = -1{,}1419$$

Therefore, the approximate value of the solution for the differential equation is $y = -1{,}1419$ for $x = 1{,}2$. What we just showed is the idea of Euler's method. Repeating these steps gives a list of values for the solution. These values are presented in the following table rounded into four decimal digits.

Table 1. Using Euler's method to approach the solution of a differential equation for $h = 0{,}1$

$\boldsymbol{n}$	$\boldsymbol{x_n}$	$\boldsymbol{y_n}$	$\boldsymbol{f(x_n, y_n)}$
0	1	−2	5
1	1,1	−1,5	3,581
2	1,2	−1,1419	3,0319
3	1,3	−0,8387	2,9004
4	1,4	−0,5487	3,0451
5	1,5	−0,2442	3,4346
6	1,6	0,0993	4,1059
7	1,7	0,5099	5,173
8	1,8	1,0272	6,8871
9	1,9	1,7159	9,8033

Theorem (Euler's Method): We take into account the problem of initial value

$$y' = f(x_0, y_0)\,, y(x_0) = y_0$$

To approach a solution to this problem using Euler's method, we determine

$$x_n = x_0 + nh$$
$$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$$

Here, h presents the size of the step and n is the full number that starts from 1. The number of steps taken is numbered by change n.

If in the example above we take the value of step $h = 0.01$, then we will win the values as in the table below.

Table 2. Using Euler's method to approach the solution of a differential equation for $h = 0{,}01$

n	x_n	y_n	$f(x_n, y_n)$
0	1	-2	5
1	1,01	$-1{,}95$	4,8328
2	1,02	$-1{,}9017$	4,6777
3	1,03	$-1{,}8549$	4,5334
4	1,04	$-1{,}8096$	4,3996
5	1,05	$-1{,}7656$	4,275
6	1,06	$-1{,}7228$	4,159
7	1,07	$-1{,}6812$	4,0514
8	1,08	$-1{,}6407$	3,9498
9	1,09	$-1{,}6012$	3,8588

Usually, h is a small value. If the value of h is smaller, then need more calculations. If the value of h if is greater, then need the less calculation

However, for the size of the largest step results in lower degree of accuracy.

With ten calculations, we are able to advance the values of the solution to the problem of initial value for x values between 1 and 2.

3. Conclusion

Euler's method is a good method for approximating the differential equation solution.

If we look at the above tables, we notice that the smaller the value of step h, the closer we get to the values of the draw solution. In case when n try in ∞, then the approximate values approach the differential equation solution.

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