UDC: 514.123:72.012
Professional paper

APPLICATION OF CONIC SECTIONS IN ARCHITECTURE

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Abstract:

In this paper, we present the equations of conic sections and the application of conic sections in the field of architecture. Mentioning some of the projects realized and under construction by famous world architects, these projects present special, unique buildings that have been built on the geometrical principles, namely the conical sections which have given special and immortal greatness, beautiful to the human eye and feeling balanced.

These mentioned projects belong to the Balkan area, respectively the state of Albania and North Macedonia. The architecture of which is affecting modernization every day.

Keywords: Conic sections, application, architecture.

Introduction

Throughout this paper, we will use a very intuitive approach to introduce and explain classical definitions of the conics and connect those definitions with the field of architecture.

Application of Conic Sessions in Architecture

Every building you spend time in—schools, libraries, homes, apartment complexes, movie theaters, even your favorite ice cream shop—is the product of mathematical principles applied to design and construction. Geometry, algebra, and trigonometry all play a crucial role in architectural design. Architects apply these mathematical forms to plan their plans or initial sketches. They also calculate the probability of issues the construction team may encounter as they bring the design vision to life in three dimensions.

Since ancient times, architects have used geometric principles to plan the shapes and spatial forms of buildings. In 300 BC, the Greek mathematician Euclid defined a mathematical law of nature called the Golden Ratio. For more than two thousand years, architects have used this formula to design proportions in buildings that look pleasing to the human eye and feel balanced. It is also known as the Golden Constant because it manifests everywhere.

The Golden Ratio still serves as a basic geometric principle in architecture. You could even call it an eternal archetype, as it evokes in human beings a universal sense of harmony when they see or stand in a building designed with this principle. In addition, perhaps not surprisingly, we see the Golden Ratio demonstrated throughout the "architectures" of the natural world. Many buildings incorporate conical sections in their design.

Architects have many reasons for using these curves, ranging from structural stability to simple aesthetics. In ancient times, architecture was considered an integral part of mathematics, so architects had to be mathematicians. Many of the structures they built—the pyramids, temples, amphitheaters, and irrigation projects—still stand.

The geometric properties of conics mixed with algebraic manipulations are used to build practical monuments and structures. They meet the practical and spiritual needs of people with their intelligent use of available space and attractive exteriors. In the modern era, architects now use even more sophisticated mathematical principles.

In the following, we will present conic sections, their equations, and their application in architecture, presenting some famous buildings around the world, which were built on geometric principles, namely conic sections, which have given you special and immortal greatness!

Definition of Cone

A cone is a three-dimensional geometric shape that smoothly tapers from a base to e flat at a point called the vertex. A cone is formed by a set of line segments, half-lines, or lines connecting a common point, the vertex, to all points on a base that is in a plane that does not contain the vertex.

TYPES OF CONES

As we have already discussed a brief definition of a cone, let's talk about its types. There are two types of cones;

1. Right circular cone

Cone, which has a circular base and the axis from the apex of the cone to the base passes through the center of the circular base. The apex of the cone lies slightly above the center of the circular base. The word "right" is used here because the axis forms a right angle with the base of the cone or is perpendicular to the base. These are the most common types of cones used in geometry.

2. Oblique cone

A cone, which has a circular base, but the axis of the cone is not perpendicular to the base, is called an oblique cone. The apex of this cone is not directly above the center of the circular base. Therefore, this cone looks like a slanted cone or tilted cone.

Conic Sections

Conic sections have been studied since the time of the ancient Greeks and were considered an important mathematical concept. As early as 320 BC, such Greek mathematicians as Menaechmus, Apollonius, and Archimedes were fascinated by these conic sections.

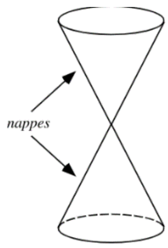


Figure 1. Cone

Apollonius wrote an entire eight-volume treatise on conic sections, in which he, for example, was able to derive a specific method for identifying a conic section using geometry. Since then, important applications of conic sections have appeared (for example, in astronomy), and the properties of conic sections are used in radio telescopes, satellite dish receivers, and even in architecture. In this paper, we will discuss the four basic conic sections, some of their properties, their equations, and their application.

Conic sections get their name because they can be created by intersecting a plane with a cone. A cone has two identically shaped parts called nappes. A nappe is what most people mean by a "cone", shaped like a party hat.

If the plane intersects both nappes, then the conic section is a hyperbola. If the plane is parallel to the generating line, the conic section is a parabola. If the plane is perpendicular to the axis of rotation, the conic section is a circle. If the plane intersects a diaper at an angle to the axis (other than 90°), then the conic section is an ellipse.

Eccentricity and Directivity

An alternative way to describe a conic section involves direction, foci, and a new feature called eccentricity. We will see that the eccentricity value of a conic section can uniquely define that conic.

The eccentricity of a conic section is defined to be the distance from any point on the conic section to its focus, divided by the perpendicular distance from that point in the nearest direction. This value is constant for any conic section and can also define the conic section:

If $e=0$, the conic is a circle

If $e=1$, the conic is a parabola.

If $e<1$, it is an ellipse.

If $e>1$, it is hyperbola.

The directrix of a conic section is a line perpendicular to the axis that defines a conic section along with the focus. The distance of the point location from the focus is proportional to its horizontal distance from the directrix and is the constant of proportionality.

Ellipse

An ellipse is a set of points in a plane whose distances from two fixed points, called foci.

If the major axis of an ellipse is parallel to the x-axis in a rectangular coordinate plane, we say that the ellipse is horizontal. If the major axis is parallel to the y-axis.

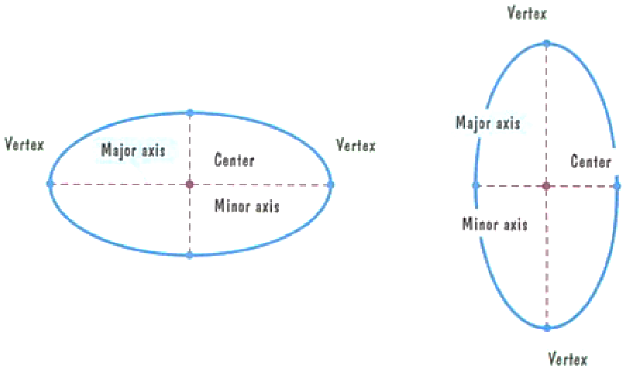


Figure 2. (a) Horizontal ellipse (b)Vertical ellipse

As can be seen from the figure a > b where a, half the length of the major axis, is called the major radius. And b, half the length of the minor axis, is called the minor radius.

The foci of the ellipse are the two reference points F_1 *and* F_2that help draw the ellipse. The foci of the ellipse lie on the major axis of the ellipse and are equidistant from the origin. The foci of the ellipse can be calculated by knowing the semi-major axis, the semi-minor axis, and the eccentricity of the ellipse.

If points F_1 *and* F_2that are foci (multiple foci) and d is a given positive constant then (x,y) is a point on the ellipse if $d = d_1 + d_2$as shown below:

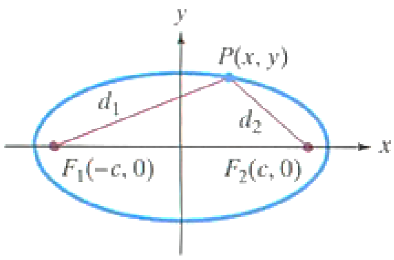


Figure 3. Focci of ellipse

The standard form of the equation of an ellipse centered at (0,0) follows:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

There are two such equations, one corresponding to the main horizontal axis and the other to the main vertical axis.

The standard form of the equation of an ellipse with vertex at (h,k) follows:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

The Application of The Ellipse in Architecture

The ellipse is without a doubt one of the most popular curves that can be seen. Even if you look around the room, you are in right now. I am sure you will see many examples of ellipses. For example, whenever you look at a circular figure at an oblique angle. The curve you see is an ellipse.

Many real-world situations can be represented by ellipses, including the orbits of planets, satellites, moons, and comets, as well as the shapes of boat keels, rudders, and some plane wings.

Nevertheless, we will stop the application of conic sections in architecture. Taking a closer look at some cases when world-renowned architects have merged architecture and mathematics!

MET Tirana, ALBANIA

Architect: Mario Cucinella

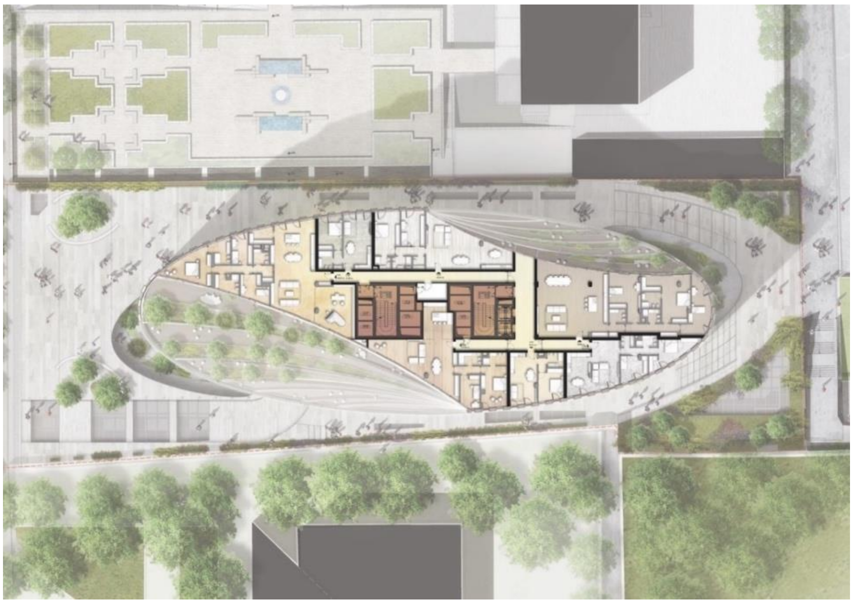


Figure 4.MET Tirana sketch

The MET Tirana Building is a new residential and office building located in one of the most important and central areas of Tirana. Conceived as if it were a new species of plant, the MET Tirana is a strikingly expressive building characterized by a certain degree of monumentality, as required by the context.

The architectural configuration transforms the constraints of urban regulations into an opportunity to experiment with "excavated" volumes. Thus, the elliptical shape of the building becomes a game of spiral terraces. The articulation of volumes, the

The composition of the facade is based on an alternation of opaque and transparent modules and the floor-to-ceiling glazing at ground level gives the building a sense of elegance and lightness.

The building is “lightened” as it goes up through a progressive subtraction of volume so that most of the elliptical plan has been removed on the top floor. This configuration made it possible to create large terraces on the upper residential floors, embellished with planters big enough for trees and large shrubs. The fully glazed walls of the apartments facing the terraces permit both generous amounts of natural light and ample views of the surrounding environment.

The architectural shapes result in the residential terraces becoming wider and more spacious on the upper floors.

Figure 7.MET Tirana

Figure 5. Met Tirana, Tower Albania

TID Tower, ALBANIA

Architect: 51N4E

The TID tower is part of a series of operations to bring the idea of the city as a collective space back into the hearts and minds of the people.

The tower is designed as a monolith, avoiding the image of the modern glass tower.

Starting from an ellipse and ending as a rectangle, the subtle transition between these two basic shapes makes a tower unique.

It does not symbolize anything but simply highlights an environmental –and as such, cultural-condition. The whole structure is straightforward and well made.

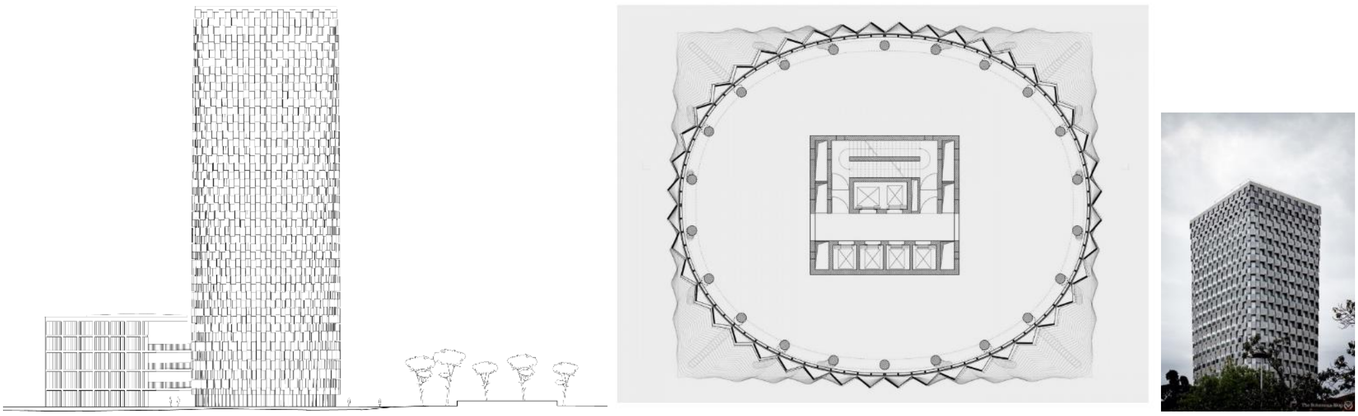
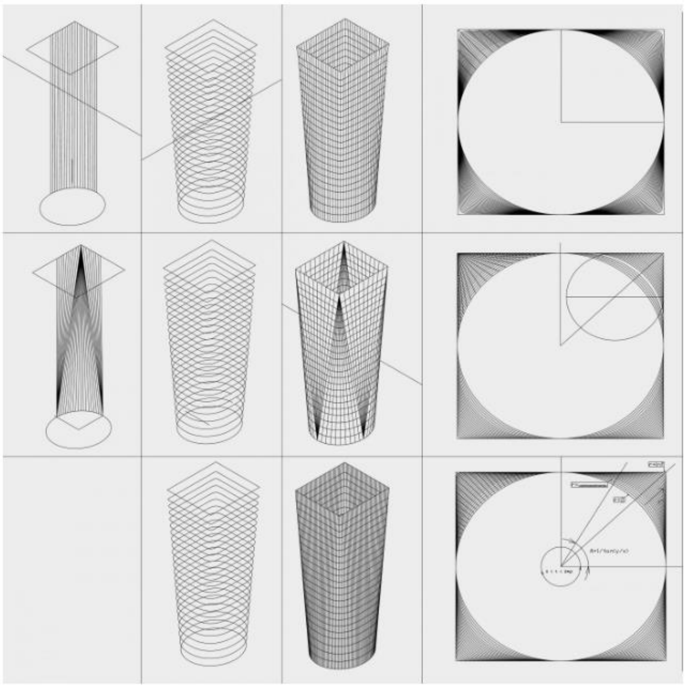


Figure 8. Ellipsoid base of TID Tower
Figure 9. TID Tower sketch



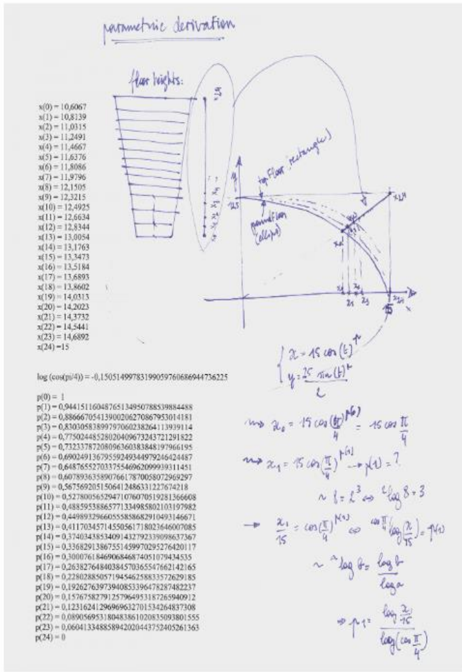
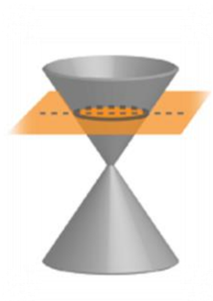


Figure 10. Tid Tower metric sketch
Figure 11. Parametric derivative calculations

2.1 *CIRCLE*



A circle is a set of points in a plane that are equidistant from a given point, called the center.

Since a circle is defined as a set of points equidistant from the center, we can use the distance formula to determine an equation of a circle. We start with a circle that has its center at the origin passing through a point and has a radius r

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{(x - 0)^2 + (y - 0)^2}$$

$$r^2 = x^2 + y^2 \text{ or } x^2 + y^2 = r^2$$

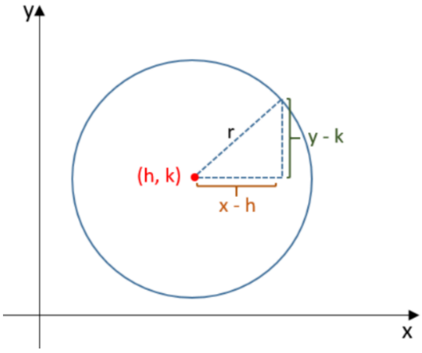


Figure 12. Circle

We have seen that the graph of a circle is completely determined by its center and radius which can be read from its equation in standard form. However, the equation is not always given in standard form.

The equation of the circle in general form follows:

$$x^2 + y^2 + cx + dy + e = 0$$

APPLICATION OF THE CIRCLE IN ARCHITECTURE

We can model many situations using circles or parts of circles. The design of arches often includes parts of circles.

The circle is a universal symbol with a wide meaning. It represents the notion of wholeness, focus, infinity, unity, eternity, the Sun, the Moon, and the entire Universe.

The ancient Greeks based the design for all their empires on the ratio of basic forms and principles of geometry. Since then, architects and designers have used circles as a way to give projects their own identity.

Pyramid of Tirana, ALBANIA

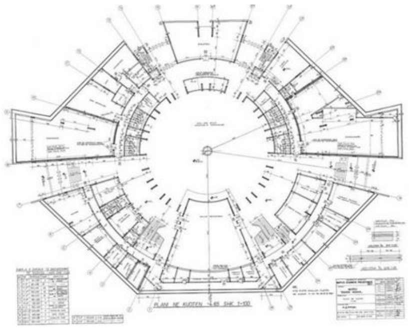


Figure 14. Base sketch of pyramid

Architect: Pranvera Hoxha and Klement Kolaneci
Retouch: MVRDV
The pyramid has a unique silhouette and a strong presence in the urban environment. The inclined facade of the Pyramid created an illusionary perspective, a unique feature at that time.

Figure 15. (a) New project of pyramid/outside
(b) New project of pyramid/inside
(c) New project of pyramid/outside

The place-mend of the glass windows follows a radial composition around a central axis of rotation. The architectural volume rises 21 meters in height but appears lower due to the inclined planes throughout its exterior and a series of platforms and stairs that lead from street level to the Pyramid entrance, that allow human scale to prevail.

After many years, a project to redesign the Pyramid has been confirmed. It will look exactly like the attached photo.

This pyramid presents a circular conical cut, which makes it special especially, in the interior, where the circular space formed by its base will be emphasized, giving it a special interior.

2.2 *HYPERBOLA*

A hyperbola is the set of points in a plane whose distances from two fixed points, called foci, have an absolute difference that is equal to a positive constant. In others words, if the points F_1 and $F_2$are foci and d is a positive constant given then.

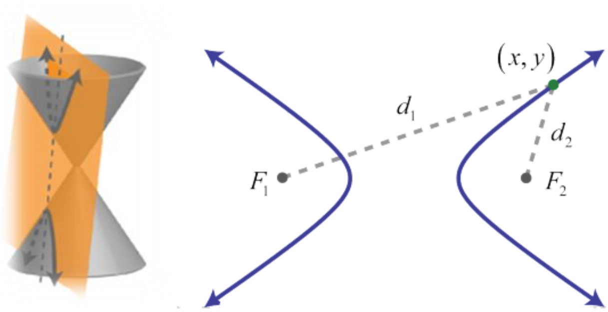


Figure 16. Hyperbola

It consists of two separate curves, called branches. The points on the separate branches of the graph where the distance is at a minimum are called vertices. The midpoint between the vertices of the hyperbola is its center. Unlike a parabola, a hyperbola is asymptotic to certain lines drawn through the center.

As they get larger, the two branches of the graph of a hyperbola approach a pair of intersecting lines, called asymptotes. Asymptotes pass through the centers of the hyperbola and are useful in graphing hyperbolas.

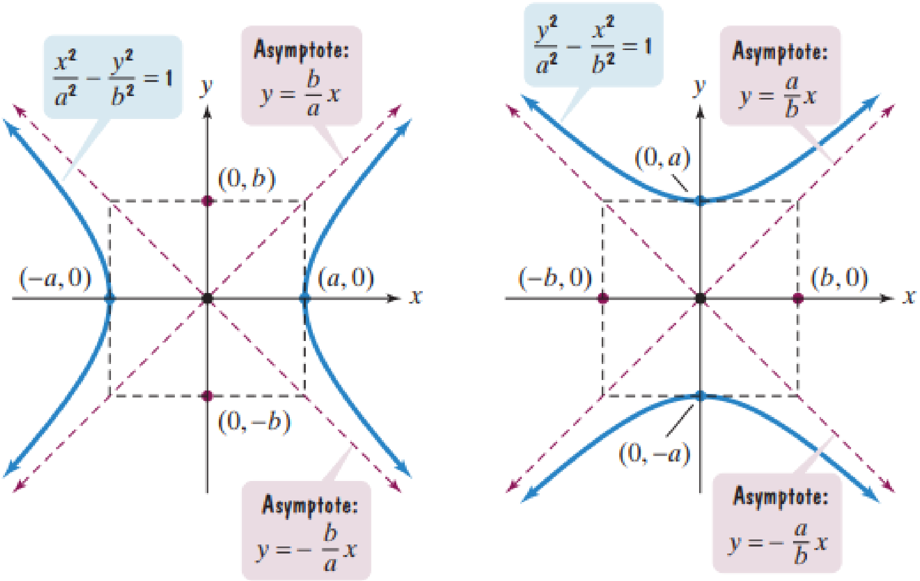


Figure 17. (a) Hyperbola opening right/left
(b) Hyperbola opening up/down

The standard form of the equation of the hyperbola with center at the origin and intercepts x (-a,0) and (a,0) follows,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \; where \; a > 0 \; and \; b > 0$$

The standard form of the equation of the hyperbola with center at the origin and intercepts y (0,-b) and (0,b) follows,

$$\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1 \; where \; a > 0 \; and \; b > 0$$

Standard forms of equations of hyperbolas centered at (h, k)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$(h \pm a, k)$$

And,

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

$$(h, k \pm b)$$

2.2.1 THE HYPERBOLOID OF A SHEET

The hyperboloid of a sheet is a surface of revolution obtained by revolving a hyperbola about the bisector perpendicular to the line (ω, ω') between the foci.
The hyperboloid of a sheet can also be obtained by rotating a hyperbola about its (ω, ω') nontransverse axis. Each point of the generator describes with its revolution a parallel circle.

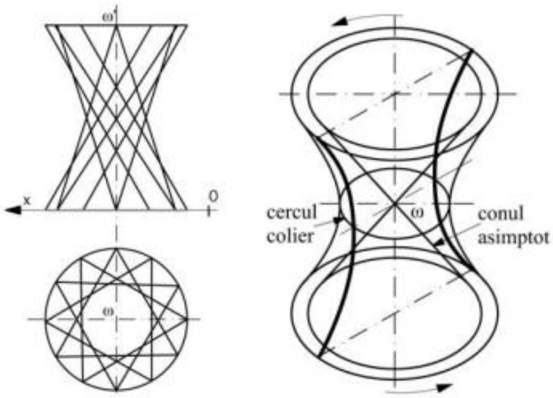


Figure 18. Hyperboloid of a sheet

The geometric conditions of the hyperboloidal surface are defined with a set of variables in the parametric model. According to the parameters h, r and n, the xyz coordinates of the nodes in the lower and upper base curves are found.

These are named as X_{ni} ,Y_{ni}, Z_{ni} and X_{ni}` ,$Y_{ni}$`, Z_{ni}`

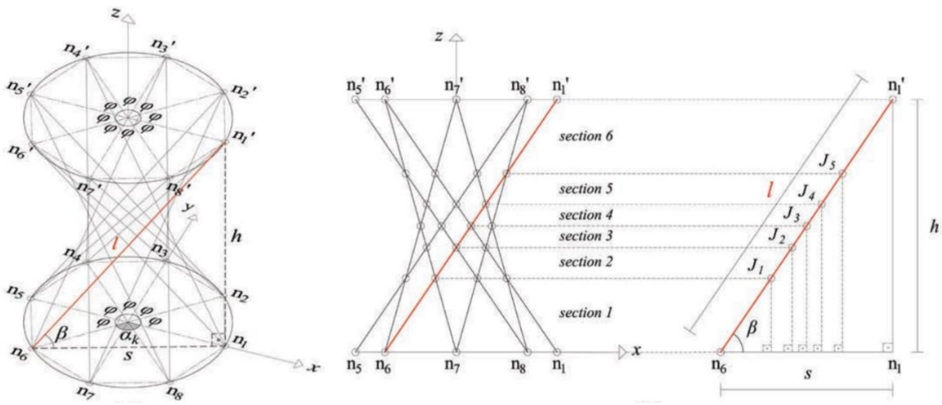


Figure 19. Parametric model of hyperboloid of a sheet

Respectively. because the curve of the lower and upper bases is identical, the x- and y-coordinates of the nodes are the same, but the z-coordinates are different. The number of intersections depends on the parameters n and α_k.

APPLICATION OF HYPERBOLA IN ARCHITECTURE

The main advantages of hyperboloid grid structures based on their structural space and their efficient construction attracted the attention of modern architects.

Twenty-first-century architects are challenged to create forms and structures that are more dynamic and adaptive. Instead of relying on conventional structures, they have explored new structural systems that can provide more innovative solutions for architectural applications.

Hyperbolic structures have a negative Gaussian curvature, meaning they curve inward instead of curving outward or being straight. They are superior in resistance to external forces compared to "straight" buildings.

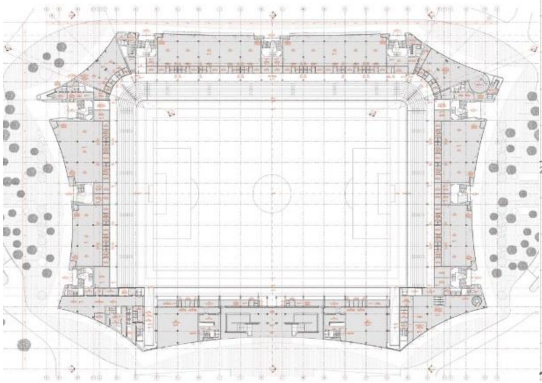
Associati's National Arena - Albania's New Stadium, ALBANIA

Architect: Archea Associati

The stadium located in the center of Tirana, in its 43,000-square-metre construction area will include a large range of facilities.

The Air Albania Stadium has been designed with a modern and dynamic profile, which inserts relaxed shapes and curved lines to create an engaging effect. The concave counter-curves of the four sides of the perimeter insert the stadium into the square and make it spacious.

The characteristic of this formed shape is that the opposite sides of the octagonal shape form hyperbolas. What characterizes this building and makes it unique as seen in the photo. The center of this hyperbola appears in the center of the sports field, that is, in the center of the stadium!



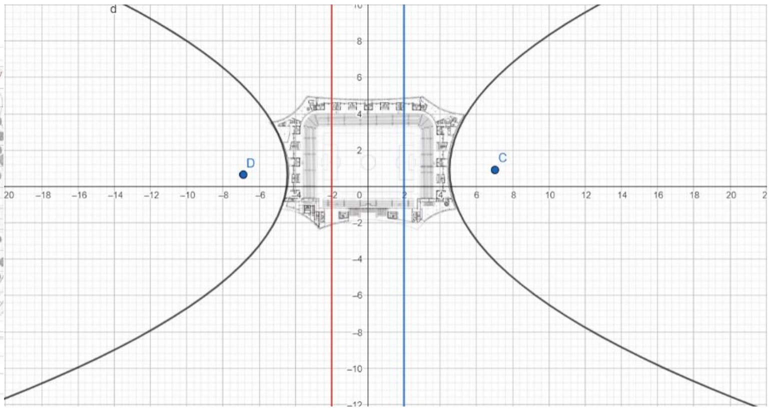


Figure 20. Air Albania sketch
Figure 21. Hyperbola formed in the construct/w/Geogebra

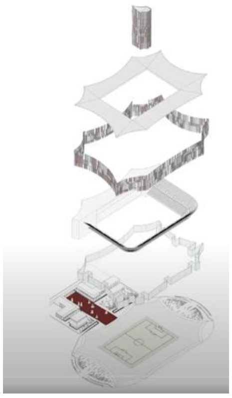


Figure 22. Layers of construction Air Albania

On the northwest corner of the stadium stands a 100-meter-high tower.

The new stadium features the presence of three stands, instead of four,

and look like a kind of classical theater.

The Air Albania Stadium, as a latest generation stadium, is characterized by a naturally more performing design, eliminating the elliptical plan.

Figure 23. (a) Project Air Albania
(b) Project Air Albania completed

2.2.2 Skenderbeg square, NORTH MACEDONIA

Architect: BINA, Besian Mehmeti Architects, QB Arkitektura

Figure 24. (a)(b) Hyperbolic construction on Skenderbeg Square

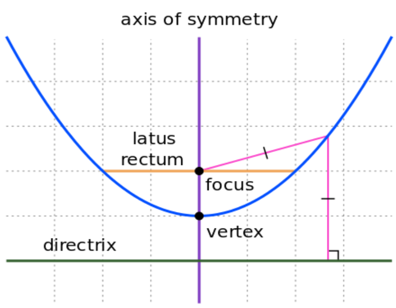
In this large square where the statue of Skanderbeg is also located, there are also several constructions of another level of modernization, these constructions present the shape of hyperbolic paraboloids that make this construction very special and modern, inside which there are green areas and walking paths for pedestrians.

The integration of conical sections by the architect in this project is truly impressive.

Figure 25. Hyperbolic construct sketch

PARABOLA

A parabola is the set of all points whose distance from a fixed point, called the focus, is equal to the distance from a fixed line, called the directrix. The point halfway between the focus and the directrix is called the vertex of the parabola.



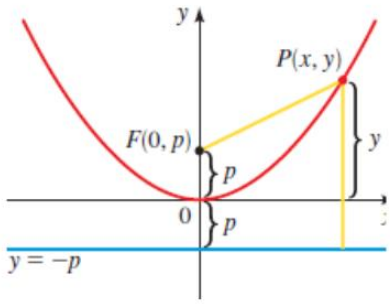


Figure 26. Parabola
Figure 27. Latus rectum of parabola

The "latus rectum" is the chord of the parabola that is parallel to the directrix and passes through the focus

The standard form of the equation of a parabola with vertex at (h,k)

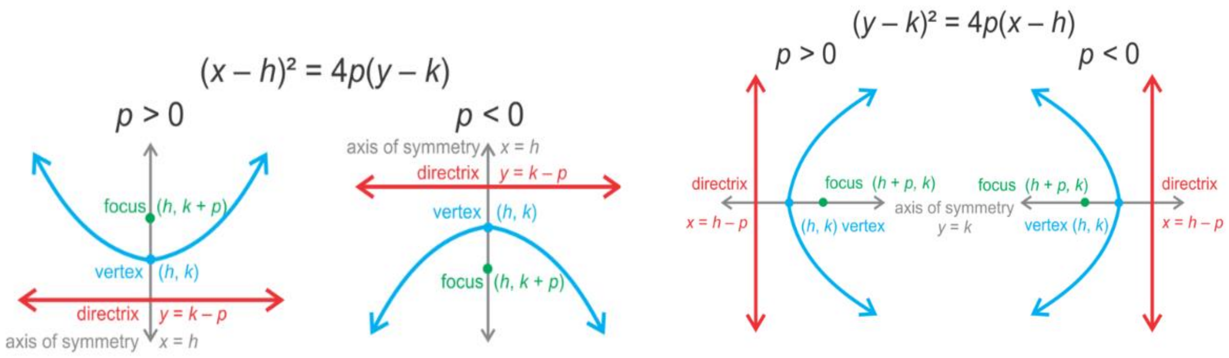


Figure 28.Parabola functions depending on the size of p

THE APPLICATION OF THE PARABOLA IN ARCHITECTURE

Parables for the architectural world are of great importance, known and used since ancient times. Parabolas are often rotated around a central axis to create the curved shape used in building designs. One of the most common architectural structures inspired by parabolas is the Arch. Arches are used in structural engineering to span an opening and support loads from above.

Examples of famous buildings that apply parabolic structures are the Arc de Triomphe in Paris, France, the Gateway Arch in St. Louis, United States, "Rua Augusta" in Lisbon, Portugal and L'Oceanografic in Valencia, Spain.

Architecture has been indebted to geometric surfaces since ancient times, as descriptive geometry books testify: "*Every architectural creation is geometry*."

However, such geometries have evolved from classic surfaces to those designed with the help of new disciplines.

Toptani Shopping Centre, ALBANIA

Architect: Winy Maas, MRVDV

The Toptani Shopping Centre is part of the recent developments in the Albanian capital Tirana. The focus exceeds expectations.

The building seen from above has an amorphous shape, like a puzzle, and vertically along its facade combines regular geometric shapes.

Including in the frontal part a symbolic parabolic curve, which on each floor has a shorter length starting from the top to the bottom. Moreover, three parabolas on its other sides make the architecture of this building very unique and impressive.

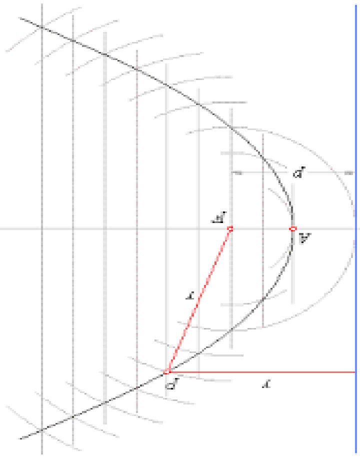
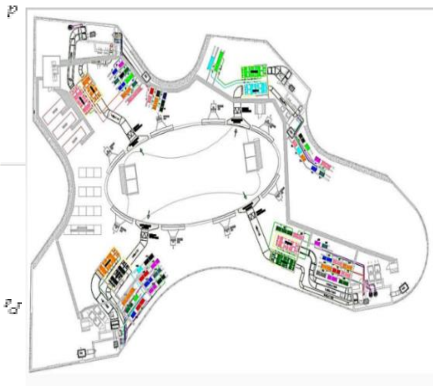




Figure 29. Toptani

Figure 30. Parabola on one side of the structure

In addition, this building includes parabolic shapes inside it, including long halls surrounded by walls in special parabolic shapes.

Figure 31. (a)(b)(c). View of the parabolic shapes inside the structure

CONCLUSION

In conclusion, conic sections are fundamental geometric shapes that have unique characteristics and are used in a variety of mathematical and real-world applications. These shapes have been used since ancient times and are used a lot even today and I think they will continue to be used continuously by architects all over the world, biased by modernization.

The countries mentioned in the paper are always moving towards modernization in addition to the framing of mathematics in it, thus creating something unique in the world that we have the honor to analyze and connect with the part of mathematics.

Understanding conic sections is essential for anyone studying mathematics, physics, engineering, or any other field that uses these shapes, and I hope that this work will help you to be actively engaged in the process of *discovery*, *reflection*, and *creation*.

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