

# STUDY OF TIME SERIES FOR ECONOMIC FORECASTS USING MULTIFRACTAL ANALYSIS

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## Abstract

In this study, we will analyze the dynamics of time series with the idea of predicting the trend of an economic variable covering intervals in different periods such as the interest rate and the exchange rate. The time series under study is the exchange rate Euro/Lek, Dollar/Lek, the basic interest rate for the local currency (Lek), and the birth rate in Albania obtained by INSTAT and the Bank of Albania with monthly frequencies. This study contributes to the field of time series forecasts in Albania and modeling using multifractal analysis. With the created models, complete modeling will be performed between the series, offers, and demands that are assumed to report changes in external influences and the effects of market forces. In addition to parallelism, non-linearity of the behavior of the data series, and the specifics of the system itself, we understood to conclude on some features of the time series. So, the daily series of the Lek/USD exchange rate is less stationary than the Lek/Euro. The evidence here supports the idea that the continuous injection of foreign exchange is among the dominant factors for the current level of the exchange rate level, but it is closer to the natural mean. Time series analysis was determined to be performed without considering the influence of external factors. This probably had its effects on the results of the models.

*Keywords:* time series, modelling, prediction, exchange rate, log-periodic.

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## 1. Introduction

Exchange rates are complicated response functions of a broad set of causes such as price indexes, employment, structures of capitals in both countries, government activities, consumptions, and many others. A general model and relationship with economic variables is presented in [3]. In [2] is analyzed a model of exchange rate, which combines international market segmentation by imperfectly competitive firms and local currency price setting called price-to-market PTM. Therein, high volatility in the exchange is expected. To some extent, the Albanian economy has elements classified as PTM and those models can be applied to the currency exchange policies.

The exchange rates in Albania has some specific characteristics. By the regulation of the Central Bank [1] and other normative Acts the exchange rates of national currencies with other currencies are considered as an important parameter of the country's financial and economic system. The statement of the Bank declares that it does not interfere with the exchange rate except if it is necessary to keep other parameters in the accepted scenario. Therefore to keep other parameters in control, the exchange rates are under control and pressured. Bank intervene to keep balances of the foreign currency and inflation in accepted levels. Clearly the bank figures out the boundary of the exchange rate. However, between limitations of the Central Bank, the process of exchange can develop freely form secondary bank and either in informal market. Another limitation or

specifics of the system consists in absence of formalized financial market in the country. In the data record perspective the system offers only daily records so the real behavior disappeared under this black out of more tiny time records. The effect of this characteristics has been addressed in our previous comments [2]. It is important to underline that Central Bank really does not totally control the ER dynamics, but only tend to keep control over bad scenarios. Therefore its role in a model could be equalized with the one of an important stakeholder and therefore the day dynamics in long term would represent the real behavior of the market.

*1.1. Useful statistical tools for stationary states and global analysis:* The stationarity scrutiny for a pdf function can be realized by performing standard stability Levy analysis. Additionally, a special class of functions called q-Gaussian

$$\rho(x) = \frac{\sqrt{q-1}}{\sqrt{3-q}} * \frac{\Gamma(\frac{5-3*q}{2*(1-q)})}{\Gamma(\frac{2-q}{1-q}*b)} * \left(1 - \left(\frac{1-q}{(3-q)*b^2} * ((s - \mu)^2)\right)\right)^{\frac{1}{1-q}} \quad (1)$$

can be very useful. It has been introduced after employing the Tsallis idea for nonadditive entropies, and later following the q-CLT arguments [4], [5]. For calculation purpose the following form is more suitable

$$G_q(x) = \sqrt{\frac{1}{3-q}} \sigma_q \frac{1}{Z_q} \left(1 - (1-q) \left(\frac{x-\mu_q}{2\sigma_q}\right)^2\right)^{\frac{1}{1-q}}, \quad 0 < q < 3 \quad (2)$$

The quantity  $d=q-1$  designates a direct measure for stationary [6]. Q-Gaussians is specific case of q-exponential  $e_q(x) \sim (1 + (q-1)bx)^{\frac{1}{1-q}}$  for positive argument. For  $q \rightarrow 1$ , classical forms are obtained. If  $q < \frac{5}{3}$ , the distribution is stationary, for  $\frac{5}{3} < q < 2$  the distribution is non-stationary and has negative variance ( $\sigma_q < 0$ ), and for  $2 < q < 3$  variance is undefined, whereas for  $q > 3$  the q-Gaussian is not a distribution object [5], [6]. Therefore, by carefully identifying the q-Gaussian fitted to the optimal histogram characterizing the data set, we can also measure the distance from the stationary by the parameter  $d=q-1$  and we can recognize the meaningfulness of the measurement itself.

The stationarity metrics provided by the q-Gaussian fit make them very useful in the analysis of heterogeneous and highly volatile time series. For a given time data series, we can span the time windows to search for the q-parameter in the desired range  $q < 1.67$ , before using any measurement instrument. Interpreting a given model as a measurement instrument, we use this preliminary analysis to discuss the legitimacy of the model itself, providing that standard techniques would have rejected the stationarity. The key is in the difference between  $q-1$  and the classification of q-values according to the q-Gaussian distribution properties motioned herein.

When analyzing nonlinear series, looking for factors after a known response, etc., the evidence of the trend underlying the dynamics is very important. If detrending procedures do not conclude, we propose the use of the empirical methods again. If the time data series is highly nonlinear and nonstationary, a good start is to explore time windows in the series where statistical properties are not fully eclipsed by dynamical irregularities. Physically we are diagnosing the system if there is a regime or near to stationary process amid transients' ones. Those "underground regularity" amidst abundant irregularities signifies the presence of regimes and trends but cannot be discovered by denoising because the signal is too complex. Herein, the Empirical Mode Decomposition (EMD) method introduced in [7] and elaborated in [8] and [9] is very useful. The non-stationary signal is decomposed in the intrinsic modes (IMF) by a spline-interpolation for the points

$$x(t) = \frac{(X_{\max,t}^{local} - X_{\min,t}^{local})}{2} \text{ giving a discrete Fourier -like form}$$

$$X(t) = \sum_{i=1}^n IMF_i + \varepsilon(t) \quad (3)$$

Particularly, the last IMF mode describes the trend of the series. In application, after evidencing the trend, we could judge if the information provided by the data series is sufficient for modeling, or we can use it to identify regimes. So, if the trend looks like a fragment of the cycle, the models are likely not reliable, and forecasting or prediction would fail. However, recognizing the underlying trend is valuable information for the system under investigation. Additionally, we proposed using the EMD to identify possible regimes in the time data series.

*1.2. Multifractal analysis on assisting empirical modelling:* Let's consider the handicaps including the system features that proscribe a classical measurement or devalue a modelling tentative. We will be dealing with the heterogeneity issues, which could be related to the signal/noise relationship and their specifics. To avoid detailed analysis following the validity for a modelling procedure, deep specific analysis, etc., we propose to provide initially a heterogeneity discussion and evidence. If we cannot correctly measure a parameter, we can try to know the heterogeneity characteristics. In this respect, the self-similarity and scaling (that lead to non-stable distributions, mostly power laws), repeatability of local events, persistence, intermittences, etc., can be helpful. We may consider the measure of that dynamical behaviour as information about nonlinear processes and use multifractal analysis to reveal it. The key mathematical objects, in this case, are fractal and multifractal parameters represented by Hurst and Holder exponents and multifractal spectrum. Just for introduction, we are briefing elementary concepts of them. The nonlinear time series  $X(t)$  with stationary increments, obeying a scaling form  $\langle |X_{n+\tau} - X_n|^q \rangle_n \sim \tau^{qH_q}$  are known as multifractal structure. Here,  $\tau$  is the time lag and  $q > 0$  is a parameter, and  $\langle \dots \rangle$  designates averaging for all (n), see [10], [11], [12]. The Holder exponent  $H_q$  or singularity exponent (the generalized Hurst exponent) describes the degree of singularity around point (t). For those structures the multifractal spectrum given by

$$f(\alpha) = q(\alpha - H(q)) + 1 \quad (4)$$

is very useful for analysing the dynamics, [28], [30] Theoretical spectral power  $f(\alpha)$  has a quadratic form, so the smoothness of  $f(\alpha)$  can be considered as a visual display of continuity of the scaling behaviour, and the un-smoothness, indicates heterogeneity. If Holder exponent is unique the structure is known as mono fractal obeying the simple scaling rule  $X(at, \omega) \sim a^H \cdot X(t, \omega)$ , [10], [11] and H is known as Hurst exponent. In this case the autocorrelation function reveals H parameter as a correlation measure from the relation  $\rho = \frac{\langle X(t+\tau)X(t) \rangle}{\langle X(t)^2 \rangle} = 2^{2H-1} - 1 \equiv 2^D - 1$ . Here, D is known as correlation index or as fractal dimension. In this sense, different scaling of large and low fluctuations can be viewed as another measurable quantity on series.

If  $\alpha_{max}$ ,  $\alpha_{min}$  are the values of vanishing ( $f(\alpha_{min,max} = 0)$ ), the quantity,

$$\Delta\alpha = \alpha_{max} - \alpha_{min} \quad (5)$$

named multifractal spectrum width” measures the abundance of scaling exponents  $H_q$ . Larger width value indicates dominant multi-scaling, and smaller width corresponds to mono-fractal structure or unique scaling. Also, the weight of smooth or abrupt local fluctuations can be read from the multifractal skewness  $skew = \frac{\alpha_{max} - \alpha_0}{\alpha_0 - \alpha_{min}}$ . Left-skewed multifractals (skews < 1) have more fine-structures in large fluctuations. Right-skewed multifractals have more fine structures in small fluctuations. The multifractal asymmetry parameter defined as

$$a_s = \frac{\Delta_{Left} - \Delta_{right}}{\Delta_{Left} + \Delta_{right}} \equiv \frac{\alpha_0 - \alpha_{min} - \alpha_{max} - \alpha_0}{\alpha_{max} - \alpha_0 + \alpha_{max} - \alpha_0} = \frac{\Delta\alpha}{\Delta\alpha - 2\alpha_0} \quad (6)$$

measures the behavior of local fluctuations. Negative asymmetry indicates a low fractal exponent of small weights and positive asymmetry. It means that extreme events play a major role. Positive asymmetries indicate

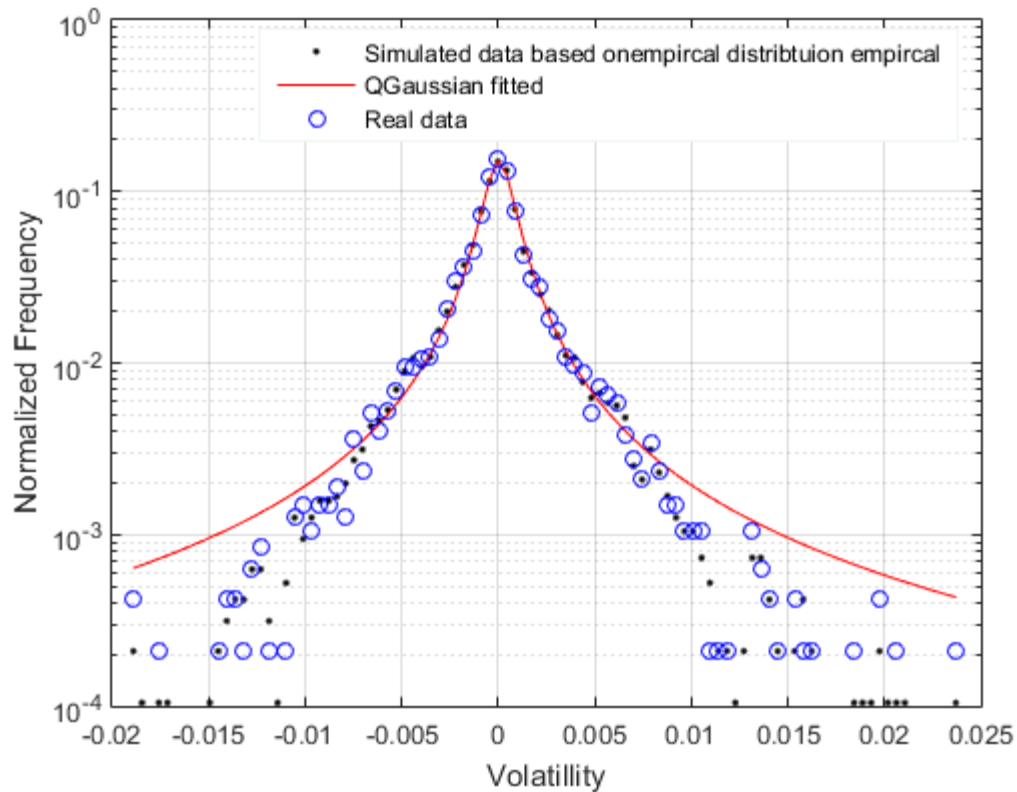
the presence of fine structure. By using equations [7] and [8] we can estimate the grade of complexity, heterogeneity in the structure, self-similarities, etc., that are valuable information for the dynamics. Also, considering the Tallies formalism that interlinks multifractal properties with q-indexes,  $\{q_{entropy}, q_{sensitive}, q_{relaxation}\}$  a more physics approach to the relaxation process is very helpful. So, the q-stationary in [13] and [14] is known as  $q_{entropy}$  providing that it resulted from the optimization of the distribution regarding to the non-additive Tallies entropy in the form  $S_{A+B} = S_A + S_B + (1 - q_{entropy})S_A * S_B$ . The quantity  $q_{sensitive} = 1 - \frac{\alpha_{max}\alpha_{min}}{\alpha_{max} - \alpha_{min}}$  which can be easily calculated from multifractal structure, measures the q-entropy production [20]. From the general point of view, the structure of the data is considered empirically as a physical coat, so, without providing detailed evidence or explanation, we proposed to use the same methodology as in a pure physics system. It is not a novelty because all complex systems are considered in such a way, but we need to enforce this treatment for specific cases where the heterogeneity is high, there are few data records in disposals, etc. In the following examples, we considered the dynamics of the dynamical features as physical property and hence a measurable observable.

## 2. Long term return distribution

The official data for exchange rates have been collected from Bank of Albania and from Oanda. We estimated the stationary of the volatility state in the long term by making use of q-Gaussians. To avoid any hidden error in the estimation of the distribution we performed bin optimization according to Scot's rule, Friedman Diatonic, and Knuth. We observed that the distribution keeps its parameters for each choice. Based on the analysis in [15], [16], and [17] for non-stationary distributions we tried the fit of a q-Gaussian as given in [18] and [17] to the histogram.

$$\rho(d) = \alpha \left( 1 - (1-q) \left[ \frac{d-\mu}{b} \right]^2 \right)^{\frac{1}{1-q}} \quad (7)$$

The displacement value  $\mu$  plays the role of the mean in classical Gaussian as seen in [5] and therefore it can suggest a qualitative estimation of the long-term mean of the volatility. In equation (7) the difference  $q-1$  measures the distance of the distribution from the Gaussian distribution, see [15], [17].

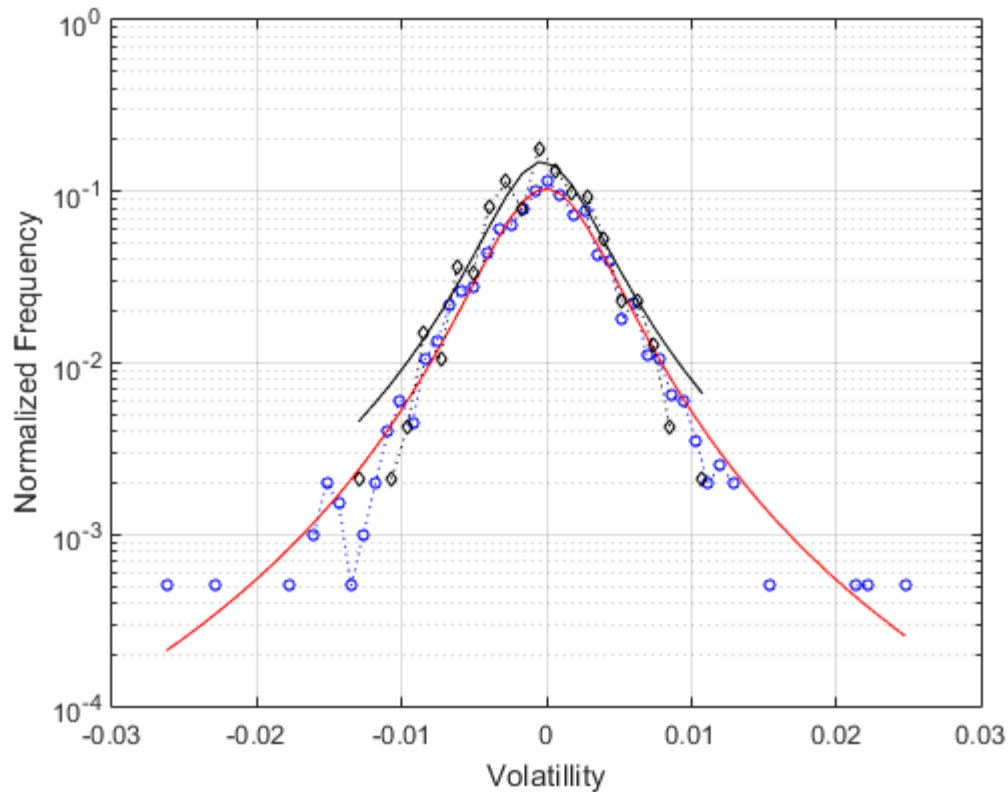


**Figure 1.** Semilog distribution for Euro returns and comparison with the empirical distribution.

By simulation of the points based on the empirical distribution we observed remarkable differences on the two sides of the volatility which indicates that the distribution of volatility has more dynamics (Fig.1). Next, from the q-Gaussian fit, we obtained  $q > q_0 \equiv 5/3$  that testify the non-stationarity of the state. In a more detailed view we obtained that  $q \sim 2.14$  so it belongs to the undefined variance according to [Tsallis]  $2 < q < 3$ . Therefore, the quasi-variance measured by distribution approach could not mathematically represent a real data variance and so does the mean. It resulted that for a long-term prediction, if inflations in EU and Albania will have the same rate, the expected scenario is. If one naively expects that the distribution would stabilize for long time which means that it would go asymptotically to a q-distribution with  $q < 5/3$ , having a negative return would be the most probable event for some more periods.

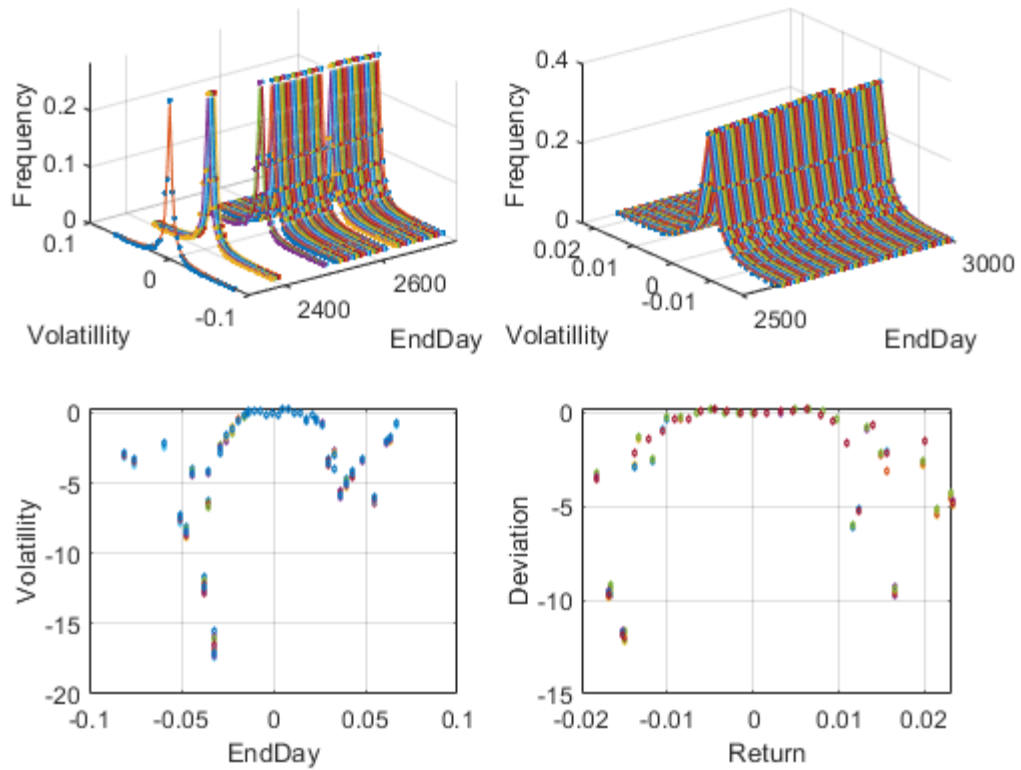
However, this behavior must be analyzed by identification of the dominant regime. Up here we can figure out that

- the state of the return is unstable
- long term state is more unstable than shorter, therefore we had a regime change in the overall trend of the exchange rate
- further analysis should be undertaken for shorter intervals.



**Figure 2.** Semi log representation of the distribution for dollar volatility.

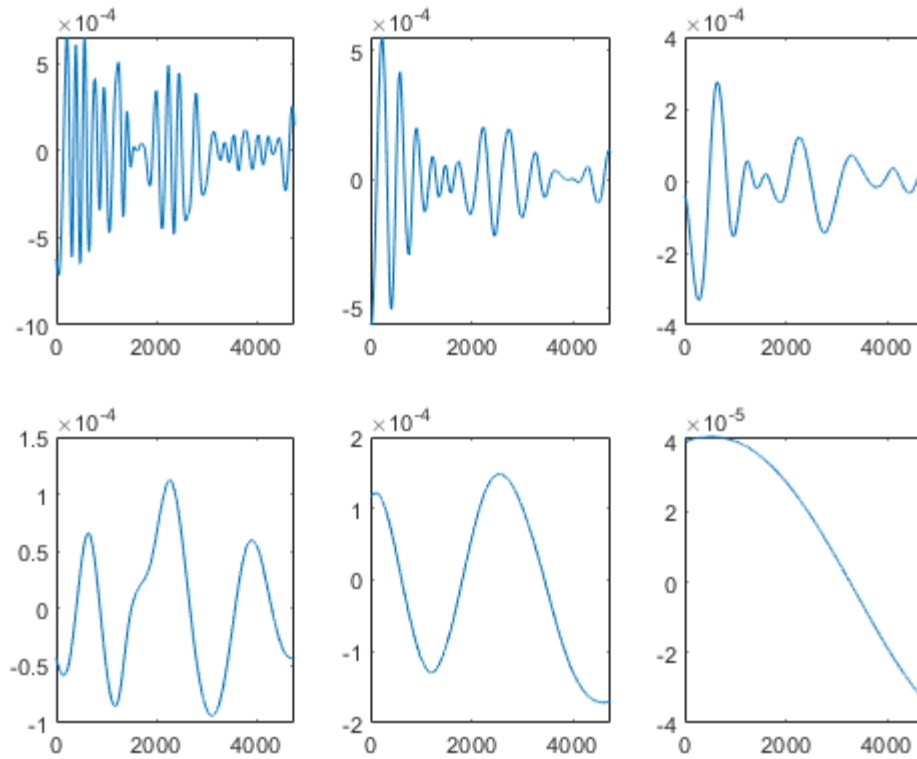
The parameter  $q$  for the period 2010-2020 is about 1.54 which belongs to stable distribution. Shorter periods such as 2020-2023 produced unstable distribution for returns. We observe that the distributions fitted with empirical densities of volatility change their parameters irregularly, Fig.2. We asked for adjusted squared to be more than 0.995 for the qualification of the fit. We obtained that by setting the ending date of the series to a week difference and starting from January 2018, there is usually a good fit of empirical densities to a  $q$ -Gaussian but the distribution is non-stable ( $q < 5/3$ ). The mean obtained by fitting procedures is found in the interval  $[-0.059, 0.006]$  with an average value at  $-0.01033$  and the  $q$ -parameter is found in-between  $[1.691, 1.891]$ .



**Figure 3.** Distributions estimates for different starting time up to may 2022.

The fit reached acceptable values (adjusted R2 0.995) starting from 40 working weeks before the ending date. Approaching this date, distributions become less unstable. We have the last one too close to the stationary distribution limit ( $q < 1.67$ ). In the empirical view, this behavior testified that based changes acted as relaxation on the overall trend. this justifies our selection of the iteration step by 5 days. It was intended to catch the relaxation on the exchange rates due to the weekend interruption of official financial activities.

Interestingly, we obtained that the distributions show remarkable deviances around return values -0.005 and 0.0045. Those deviances are not related to the quench behavior because they are positioned far from the edge of the interval. Therefore, they contain important information. From the distribution mechanism paradigm, q-Gaussian arises from the no-additively or merely form q-additive behavior. It resulted from this point of view that other mechanism acts on the system. They can be related to Central Bank activities.



**Figure 4.** EMD for long period: 2010-2020. The trend of the volatility of the return shown in the last mode will decrease but with smaller amount and later the curves is expected to change trend (remaining part of a sine).

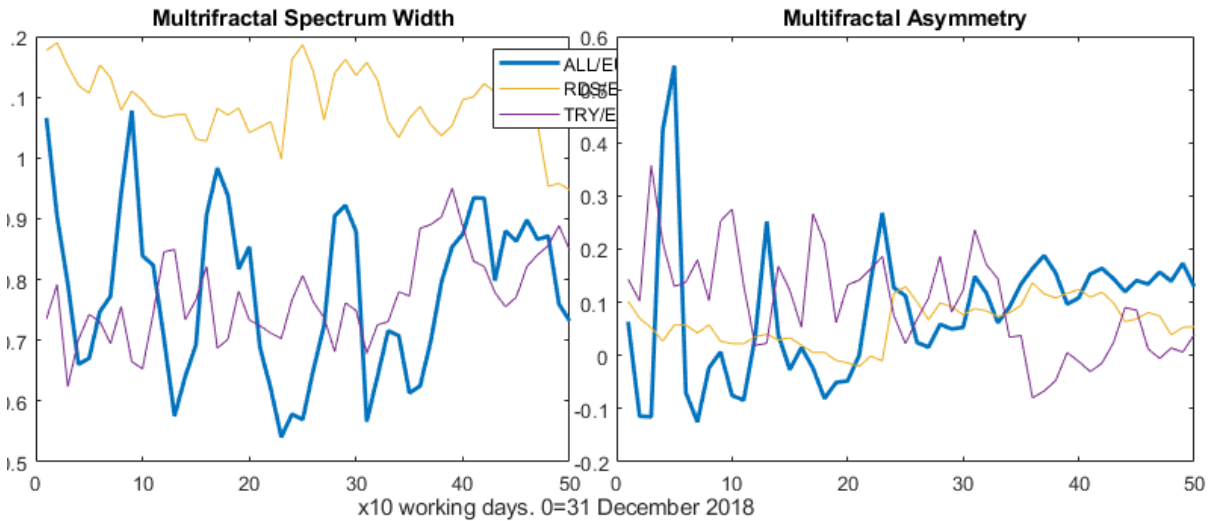
As seen on the empiric mode decomposition the volatility of the exchange rates appears to have many modes (14 in our approach) and there is no distinguished periodicity on them however, in the long-term consideration it is clear from the last mode on the fig (4) that the volatility is keeping go lower. The last mode that usually reflects the trend is in the midterm. For further comments on this approach, we decided to change the monitoring windows. We are interested in obtaining the most appropriate interval where the last mode which represents the trend of the sires, would have an apparent cycle. In this case, we can identify a possible regime by hypothesizing that the stochastic process that governs the behavior would have overlapped with this trend. To simplify the search procedures, we assumed that the changes are most likely to occur after at least a month so we chose step 23 working day. Next, we assumed that the long-term behavior is difficult to analyze because of the multiplicities of the regimes and processes on the dynamics.

*2.1. The performance of the ALL-EURO FX rate during the 2020-2021 economic crisis:* Our idea for a physics approach to the study of time series for real systems has been used in [21] and for analyzing the performance of the ALL-Euro exchange rates. From the econometric point of view, this analysis could include the volatility  $r_t = dif(\ln(x)) \cong \frac{x_{t+1}-x_t}{x_t}$  behaviour or the compatibility of the logarithmic rates of nominal exchange rate as given in [10], [11]



$$e_t = (m_t - m_t^f) - a_1(y_t - y_t^f) - a_2(i_t - i_t^f) + a_3(p_t - p_t^f) + \varepsilon_t, \quad (8)$$

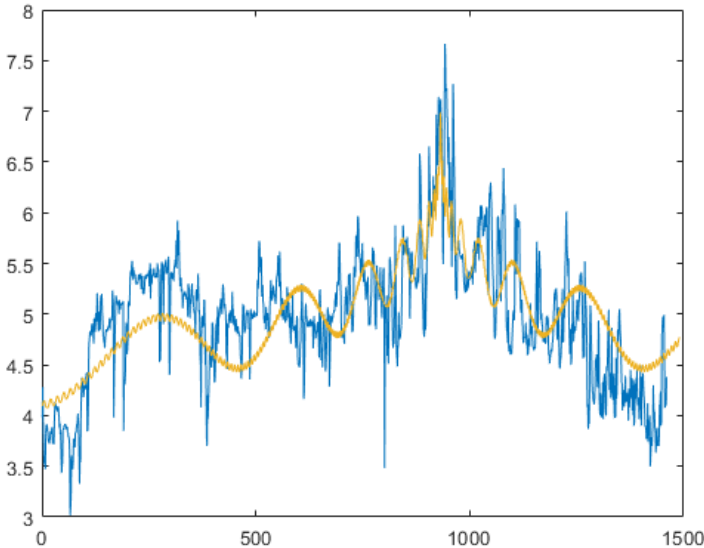
where  $m_t$  are the money supplies,  $i_t$  is the interest rate  $y_t$  represents incomes or a GDP measure for each country,  $p$  is the purchasing power ant indices  $f$  stands for “foreign”. The term  $\varepsilon_t$  counts for stochastic effects and is usually assumed proportional to a Weiner process. What makes our system specific and therefore empirical method useful are the size of the informal sector, the common use of transactions in Euro or USD directly, missing a financial market etc. Especially the terms related to the money supply  $m_t$  remain unknown due to the presence of the informal money. So, despite the usefulness of the formula [22] the ALL-EURO FX statistical and dynamic features would provide an answer for the financial performance of the country during a crisis period. So, whatsoever a policy  $P(t)$  has been, the resulting distribution of the RoR  $r_t = dif(\ln(x)) \cong \frac{x_{t+1}-x_t}{x_t}$  and the nature of its fluctuations will tell fi the measures have been satisfactory or not. In this framework, the analysis if the performance of the policy makers could be reduced on the empirical descriptive analysis regarding the stationarity of the distribution  $\rho(x_t)$ , the heterogeneity issues and measures, multifractal properties, like the above. Firstly, the  $q$ -parameter read from the  $q$ -Gaussian fit indicates that the non-stationary degree remained nearly the same for series starting from January 2019, and ending in 2020 or 2022 respectively. The  $q$ -value of the Euro/ALL FX was around 1.9~2 and the distribution of RoR has resulted amongst most stationary among RoR distributions of the Turkey Lira (TRY), Serbian Dinar (RSD), Macedonian Denar (MKD). However, the  $q$ -mean of the ALL RoR is higher than others expect TRY/EURO, indicating that the distribution analysis is not sufficient for the conclusion. We address for this purpose the multifractal analyses. Series now start in 2017 and ending on 2019+n\*10 days (every two-week working day).



**Figure 5.** Evolution of the spectrum width and asymmetry 2015-2022

The multifractal width is Large and oscillates throughout nearly three months. A light decrease of the oscillating amplitude is observed. On the other side, the multifractal asymmetry has had high amplitudes, but it has decreased by the end of 2022. Those arguments indicate that policies and market effects have interacted dynamically. This initial anxious phase has been left over time and the system has started to relax after the first year of the crisis (roughly from the graph, more than one year, about 300 working days). This initial anxious phase has been left over time and the system has started to relax after the first year of the crisis

(roughly from the graph, more than one year, about 300 working days). Similarly, we have discussed the presence of the self-organization regime for series of overnight interest rates which are another indicator of the financial performance and health. Initially, we used the series containing values form 2007. By exploring the possible critical times and cyclic frequencies using  $\log(t - t_c)$  in the role of regular time, we obtained that the Lomb periodogram has a relief shape which culminate for  $t_c = 3442$  and has a second pick at  $t_c = 3540$ , whereas the cyclic frequency is near 20. We recognized those findings as indicators that a pseudo-critic time moment exists around  $t=3442$ , which is identified with a regime change. The reining part of the critical behaviour has been developed by directly fitting the LPP form to the actual data. We observed a regular near parabola the multifractal spectrum that indicates smooth dynamics and continuous scaling.



**Figure 6.** Finding the date of regime change by double-checking the critical time

Empirically the dynamics look like an anti-bubble. The regime started as a relaxation stage after a culmination due to the coupling effect nearly two years ago, corresponding to the middle of a pandemic-inflicted crisis.

**3. Conclusions**

Using a prior analysis of distribution has Improved the analysis of the exchange rate dynamics. The period 2015-2022 is populated from short-to-medium localized regimes, some of which have self-organization properties. The last regime is likely to have started around 2015; the trend of return has reached the lowest values and has started rising. Shorter regimes are more dynamic. The distribution of the return for precious metal prices, EURO, and USD/Lek exchange rates differs significantly. Meaning different mechanism or facto represents.

In the distribution of long-term RoR of daily exchange Euro/Lek, three states can be distinguished. Small or central RoR (<-0.01, the more stable state. High positive RoR>0.01, stable state. Next, we evidenced that the dynamics of the multifractal features for data series of the FX ALL to EURO has been intensive at the

beginning of the crisis 2019-2022 and has smoothed significantly toward the end of 2022. Also, the overnight interest that entered a regime near the middle of a crisis, has remained in it following an anti-bubble behavior. The combination of empirical methods here is seen as a solution for similar cases where standard models fail or are inapplicable do to specific influences, sparse data, or heterogeneous series.

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