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Professional paper

FORM-FINDING OF ARCHES

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Abstract

Parametric design should not be understood as the use of computers to design and manipulate architectural form in new freeform shapes. The algorithms that guide parametric design allow architects to overcome the limitations of traditional CAD software and 3D modelers, reaching a level of complexity and control which is beyond the human manual ability. Algorithms-Aided Design presents design methods based on the use a traditional or visual algorithm integrated with a the 3D modeling software allowing users to explore shapes that are defined by specific parameters.
This research focuses on strategies that help us shape forms by using forces (loads) as the guiding parameter. It explores different approaches of implementing these parameters in the Rhinoceros modeling software through the Grasshopper virtual programing environment that enable us to simulate the behavior arches and form-find the optimal shape under varying conditions.
The research shows that accurate simulation of physics is possible in the environment and that with this kind of form-finding approach the design is always focused on structural optimization. Form-finding defined by force rather than typology allows designers to create new structural solutions, which are not only spatially complex and always constrained to be in static equilibrium, but also free from any prior biases towards known geometries or typologies.

Keywords: parametric design, form-finding, force, Hooke's law, catenary, simulation

1. Introduction

Traditional drawing is an additive process, in which complexity is achieved by adding and overlapping independent elements (signs) transferred to paper. The associative links between the elements of the drawing cannot be established and the consistency of the drawing is not guaranteed by the medium but by the architect. In this case, the drawing is not a smart medium, but an abstract representation of reality, which is based on standards and conventions. The logic of adding to the traditional drawing implies certain limitations. The drawing process excludes the physically relevant aspects that drive the creation of form in the real world. Traditional drawing cannot manage forces, such as gravity, and constraints that affect and limit deformations and displacements. The limitations of the drawing process do not allow new ideas to be explored and limits designers to pre-defined and known construction systems.
Through the digitization of the drawing process, these limitations are not immediately overcome by the application of the computer alone: computer-aided design (CAD) software simply improves the ability to perform repetitive tasks without affecting the design method. Similar to traditional drawing, CAD relies on the user (architect) to determine overall consistency by adding digital characters or geometric primitives to a digital sheet/space and controlling CAD layers; whereby this method can be seen as a translation of additive logic in the digital space. Despite the limitations, drawings represent a stable working method of architecture throughout history, but this is possible because architects rely on typology, the use of well-proven, preconceived solutions, and tectonic systems. Typology makes drawing not only a medium of communication but also a system that allows designers to explore and refine variations within a specific set of formal and structural constraints.

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Conventional drawing is first questioned by the emergence of a new approach through from-finding methods. These methods appear in architecture at the end of the 19th century and aim to explore the possibilities of creating new and optimized structures found through complex and associative relationships between materials, form and structures. Pioneers such as Gaudí (Tomlow et al., 1989), Isler (Isler, 1980) and Otto (Otto and Rasch, 1995; Gass, 2016) both reject typology and look at the processes of self-formation in nature as a way to organize the architectural process.

Figure 16. Reproduction of Gaudi's hanging chain model for the crypt of Colonia Güell, Barcelona (left); Form-finding using fabric and polyester by Heinz Isler (right)

Because form cannot be derived from proven solutions, traditional drawing cannot be used as a tool to predict the results of the design process, thus early form-finding relies on physical models such as: soap films, which find minimal surfaces, and suspended fabric, which finds compression-bearing arches only (Addis, 2013). In other words, drawing as a medium for exploring form is replaced by a physical model that employs analog devices that show how dynamic forces can shape new self-optimized architectural forms. However, these techniques have numerous limitations. Transferring the geometric properties of the resulting form to the scale of a building object can be quite difficult and the behavior of the material cannot always be successfully simulated through a model of much smaller dimensions (Addis, 2013). Making models for complex architectural concepts can be very time-consuming, and the model can be impractical for interaction and changes during the design process.

2. Digital simulation: Spring-particle system

The shortcomings of the physical methods of finding the form are overcome through their implementation in modern computer software for three-dimensional modeling and simulation. Through the implementation of forces as a shaping parameter in the early stages of the design, the final result of the form can be significantly improved in terms of load-bearing characteristics. The added value of the forms obtained in this way, in contrast to the arbitrarily designed ones, represents a much better basis for further development and constructive analysis, which means fewer changes in the design process and better control from the beginning.
Several approaches can be considered for simulating the behavior of cables under the influence of gravity and finding a shape that leads to pure compression or tension. The aim of the simulation is to show the behavior of rigid bodies in reality as closely as possible.

Structures that transmit forces through axial compression or tension have an increased capacity to withstand loads with smaller cross-sections. Traditional form-finding strategies for axially loaded structures involve complex physical models: suspension chain networks and membranes. The hanging chain model, a form finding technique for arches, is a physical model, made with chains that when suspended at the ends under the action of gravitational forces take a form that transmits the forces through pure tension; such a form, which is called "funicular" (Figure 2). If mirrored gives the shape of an arch which transmits forces only through compression (Hooke, 1676). The curve that is generated by a hanging chain attached to its ends that evenly distributes its own weight under the action of gravity is called "catenary" (Adriaenssens et al., 2014).

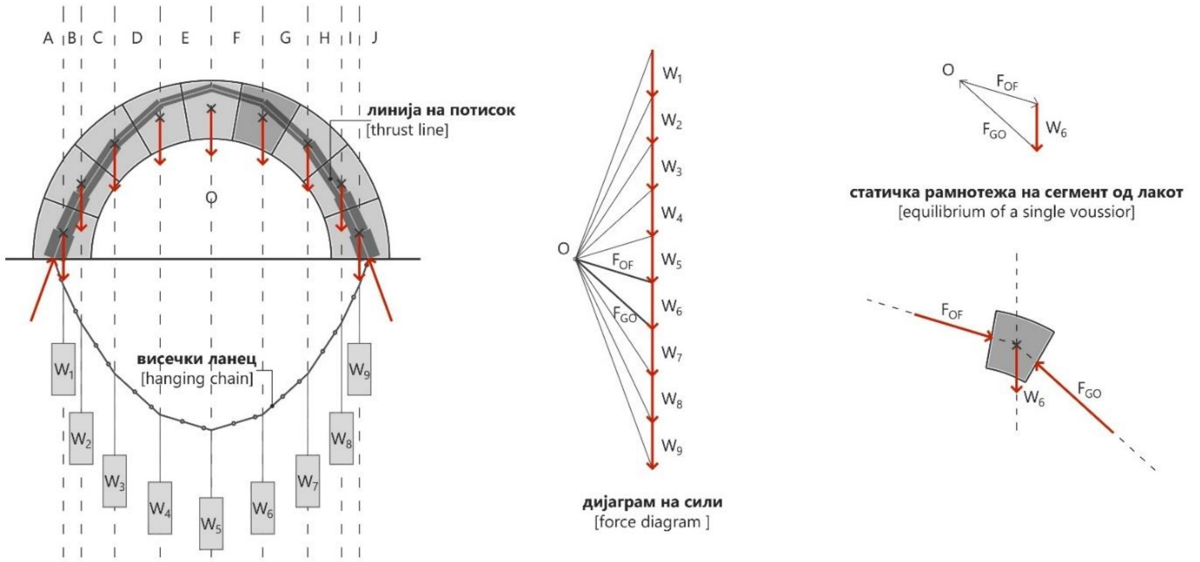

Figure 17. Finding the Shape of an Arch Hooke's Hanging Chain Model

The use of these techniques through physical models is difficult and time-consuming, which is why few designers use them in practice for finding architectural form. Traditional models today can be digitally simulated using spring and particle systems, through which the physical behavior of deformable bodies is analyzed. While traditional techniques are impractical to apply, digital simulations allow designers to explore form, in real time, by updating forces, supports and physical properties.

A spring-particle system is a discretization of a continuous model into a finite number of masses, called particles, connected by perfectly elastic springs. The main components of the spring and particle system are:

(1) Particles: Each particle in the system is a lumped mass, which changes position and velocity as the simulation evolves; (2) Springs: A spring is an elastic linear connection between two particles that behaves according to Hooke's law: a spring has an initial rest length and a stiffness value (k); (3) Forces: weights and external loads are simulated by vectors applied exclusively to the particles; (4) Supports: particles that do not change position during the simulation. Once the simulation starts, the particles move from their initial position until they reach an equilibrium state that depends on the initial geometry, force vectors, and defined spring properties. According to Hooke's law, the lower the stiffness or k value, the greater the elongation of the spring. Since the particles in this system behave as joints with no capacity to resist moments, the equilibrium solutions carry defined loads exclusively through axial forces.

Spring-particle systems, through mathematical solvers, perform iterative calculations and approach a state of equilibrium when the sum of all forces is zero. Each subsequent iteration approximates the position and velocity of the particles from the previous step and leads us closer to an equilibrium solution. This process, similar to frames in animation, creates the illusion of motion when iterations (frames) are calculated in a continuous sequence. Most software packages that simulate such systems are standalone and not fully integrated into CAD or other modeling software. Kangaroo, developed by Daniel Picker (2013), is easy to use and integrated as a toolbox for Grasshopper. The workflow in Kangaroo consists of three basic steps: discretization, establishing the system of springs and particles, and computing the system (form-finding).

Discretization is defining a deformable body, such as a flexible cable, by discretizing the geometries in Grasshopper that are defined through non-uniform rational B-spline or NURBS. Kangaroo requires NURBS curves to be converted to lines and NURBS surfaces to be converted to meshes (i.e. points and lines) because it cannot handle NURBS surfaces and NURBS curves. After discretizing the geometry, the lines are converted to springs and the points to particles using specific components located in the Kangaroo tool palette. The vectors representing the forces are applied to the particles, and the anchor points are selected by the user.

3. Cable simulation

To simulate the behavior of a flexible elastic cable suspended between two endpoints and subjected to loads caused by its own weight, a horizontal line is first defined between two endpoints. The first step is the discretization of the geometry, by dividing the line into a certain number of segments. The greater the number of dividing points, the greater the final deformation. The dividing points of the curve are the particles in the system at which loads are applied or other desired constraints are placed. The geometry processed in this way represents input information for building a system of springs and particles. Through the Kangaroo (Length) components, the segments into which the curve is divided are converted into springs. Through the Load component, a force vector is placed on each point or particle that simulates the mass. The Kangaroo solver takes the outputs of the previous components as input necessary for the calculation. By default, the simulation affects only the particles, which change velocity as the simulation tries to reach equilibrium. Once the simulation starts, the particles move in the direction of the force vectors which are constrained by the elastic-linear properties of the springs. The geometry of the cable is changed several times until it reaches equilibrium (Figure 3).

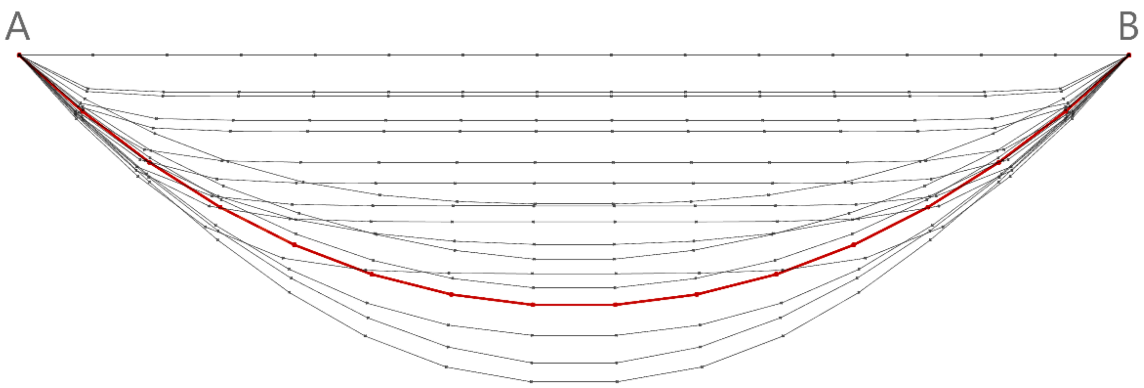

Figure 18. A display of the sequence of positions taken by a cable while the Kangaroo simulation is running. The cable bounces several times until it reaches an equilibrium state (red polyline)

4. Elastic behavior: Hooke's law

When the cable reaches an equilibrium state caused by the influence of external forces and relative to the elasticity of the springs, the length of each segment increases. The elastic behavior of the springs follows Hooke's law which states: displacements or the size of the deformation of the body is directly proportional to the acting force, i.e. the load. Under these conditions the body returns to its original shape and size when the loads are removed. Mathematically, Hooke's law is formulated by the expression:

$$F = k \cdot x \tag{1}$$

In this context, F denotes the applied force, typically quantified in Newtons (N). The parameter k represents the stiffness of the body and is generally expressed in N/cm. The variable x corresponds to the change in length or deformation of the body, such as a spring. The stiffness constant k is contingent upon the material properties and the cross-sectional geometric characteristics of the elastic body. Upon the removal of applied loads, the cable attains a length referred to as the final length, which does not necessarily coincide with the initial length. The duration required for the cable to reach equilibrium is influenced by both the stiffness and the damping constant, the latter of which is associated with the friction induced by the applied force. A higher damping constant results in a slower rate of deformation. It is crucial to note that the damping constant affects only the rate of deformation, without influencing the overall change in length.

The algorithm in Grasshopper considers the cable as a single spring; the calculated spring deformation results agree with Hooke's law. Components take into account the physical characteristics discussed in the previous example, as well as other important parameters, including:

- Connection: springs are linear elastic connections. The connection input requires lines, any other geometry will return null results. The length before loading each line is called the Initial Length.
- Stiffness: according to Hooke's law, the higher the value of k, the smaller the deformation. The stiffness is determined by the properties of the material as well as the cross-sectional area of the spring.
- Damping: the damping input affects the rate of deformation, without affecting the change in length.
- Ultimate Length: The length the spring tends to reach after the loads are removed. This parameter is essential to simulate the behavior of different materials. Three cases can be distinguished: (1) the final length is equal to the initial length the simulation mimics perfectly elastic behavior (2) the final length is less than the initial length the simulation mimics the effect of pre-tensioning the springs which determines their minimizes their length or area (3) the final length is greater than the initial length, the relaxation or elongation of the springs is simulated.
- Plasticity: the maximum elastic deformation, compared to the rest length.

5. Catenary curve simulation

Catenary curves can also be simulated using spring-particle systems. By definition, a catenary curve is the curve that an idealized hanging chain or cable assumes under its own weight when supported only at its ends in a uniform gravitational field. Therefore, the cable must meet four conditions:

- To be suspended from its endpoints;

- To be ideally elastic, after the action of the external load, the material returns to its original form;
- To be made of homogeneous material;
- If it is inextensible, the deformations are relatively small compared to the dimensions of the cable itself.

The function whose graph is the catenary curve is expressed as:

$$y = a \cdot \cosh(x/a) \quad (2)$$

The distance from the x-axis to the point on the curve with a tangent slope equal to 0 is expressed as the variable a.

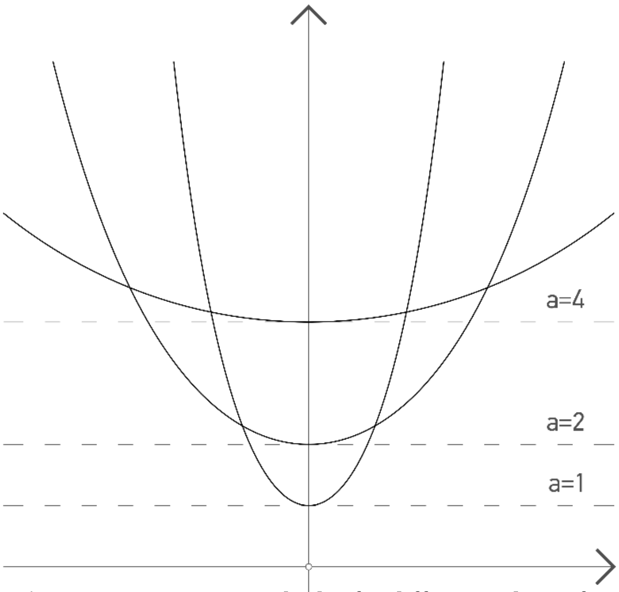

Figure 19. Catenary graph plot for different values of a

The problem arises because, even if a high stiffness value is set, the springs will not be inextensible and will undergo changes in their length after running the simulation, so the fourth condition is not met. As a result, the simulation will generate a curve that is slightly different from the catenary curve (Figure 5). For a closer approximation to the catenary curve, an arc can be used as the starting geometry, described by three points co equal to the length of the desired catenary curve

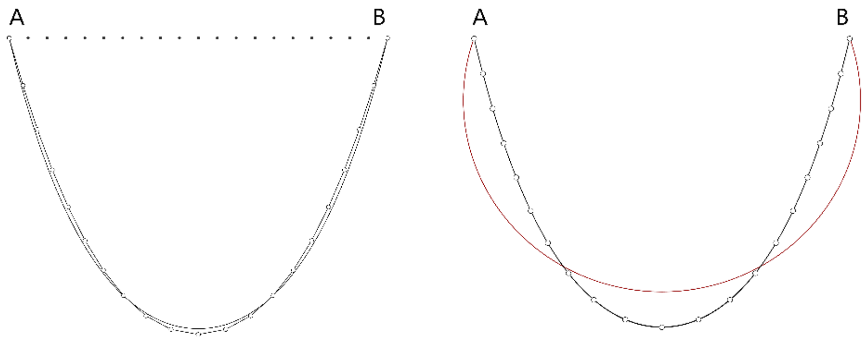

Figure 20. A line with gravity loads placed at the particles will give a curve that is slightly different from the catenary curve. A closer approximation to a catenary curve is obtained if the input geometry that is discretized is an arc

When the simulation is initiated, the segmented arc moves in the negative Z direction and takes the form of a catenary curve in its final position. The calculated polyline conforms to the four conditions of the definition and the segments change minimally in length from their initial length.

6. Further research

This principle can be extended beyond the two-dimensional arch shape and applied to shells with different geometries. Three-dimensional shapes are significantly more complex due to the possible multiple paths for the transmission of loads. Unlike a two-dimensional arch, a three-dimensional shell can carry a wide range of different loads through membrane action without generating bending stress. Analogous to Hooke's hanging chain for the arch, a three-dimensional model of cross-chains can be created to find the shape of the shell.
From a computer simulation and digital shape finding perspective this suspended model can be used to discretize a shell, in which elements are connected at nodes, and the model can be used to help define a continuous surface. Once mapped, this geometry would represent the shape of the shell stressed only in compression.
To simulate membranes or other surface materials, a two-dimensional mesh of springs and particles needs to be defined. Networks can be established using a variety of methods; the most commonly used technique is to convert a NURBS surface to a mesh. After the model is discretized, the edges and vertices of the mesh are extracted, the edges are taken as springs and the vertices as particles in which the mass is concentrated. Angular vertices are usually the positions in which bearings are defined.

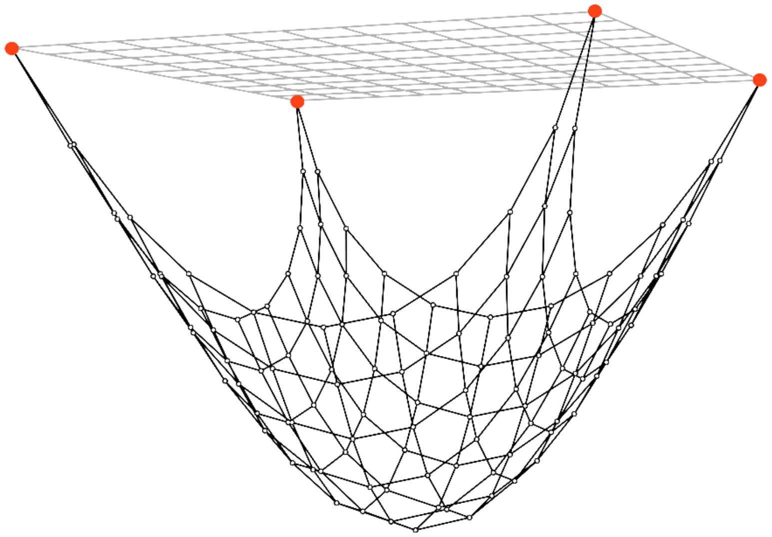

Figure 21. The shape of the shell obtained through the simulation in Kangaroo

Conclusion

Through form finding methods, design is always aimed at constructive optimization. From a conceptual point of view, this cannot result in forms, which are free-generated without any structural and constructive principle. Numerical calculation techniques replace fully experimental methods of construction design and analysis - mathematical optimization is used which, based on one or more selected criteria, uses the computing power of the computer to interactively search for optimal solutions to a problem from a range of possible solutions. This change is relevant, from the point of view of architectural design. Unlike classical form finding, the typology of the construction system no longer needs to be fixed. Optimization allows the original concept of shape finding, literally aimed at finding an optimal shape, to change into a process that can be defined as "shape improvement" - this new process is aimed at improving the performance of an already existing spatial configuration, which does not necessarily mean reaching a structurally optimal form. The parametric design process allows optimization to be included at any step of form genesis and allows free modeling by the architect which can then be refined through precisely defined parameters. A final fundamental aspect of optimization is that it is not limited to solving problems of a static nature, which is a drawback of form finding based on physical models. From a simple problem-solving tool, optimization becomes an effective tool for "exploring form" and supporting conceptual design. It pushes the boundaries of classical form-finding and defines several new research directions that completely redefine the relationship between architecture and engineering.

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