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SOFT INTERSECTION ALMOST WEAK-INTERIOR IDEALS OF SEMIGROUPS: A THEORETICAL STUDY

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Abstract

This study aims to introduce the concept of soft intersection almost weak-interior ideals of a semigroup, which extends the notion of nonnull soft intersection weak-interior ideals of a semigroup. We explore the properties of the ideal in depth. We show that soft intersection almost ideal and soft intersection almost weak-interior ideal coincide with each other when the soft set is idempotent and we also illustrate that an idempotent soft almost weak-interior ideal is a soft intersection almost subsemigroup. We also establish significant connections between almost weak-interior ideals and soft intersection weak-interior ideals of a semigroup concerning minimality, primeness, semiprimeness, and strong primeness.

Keywords: Soft set, semigroup, (almost) weak-interior ideals, soft intersection (almost) weak-interior ideals

1. Introduction

In theoretical computer science, automata, coding theory, and formal languages, as well as in graph theory and optimization theory, semigroups play a fundamental role as algebraic structures. Ideals are essential for the advanced study of algebraic structures and their applications. To further this study, it is necessary to generalize ideals in algebraic structures. Various mathematicians have made significant contributions by introducing extensions of ideals in algebraic structures.

The concept of almost left, right, and two-sided ideals of semigroups was first introduced by Grosek and Satko [1] in 1980. Bogdanovic [2] later extended the notion of bi-ideals to almost bi-ideals in semigroups in 1981. In 2018, Wattanatripop et al. [3] proposed almost quasi-ideals by combining the ideas of quasi-ideals of semigroups and almost ideals. Building on the concepts of almost ideals and interior ideals of semigroups, Kaopusek et al. [4], in 2020, introduced almost interior ideals and weakly almost interior ideals of semigroups, examining their properties. Subsequently, Iampan [5], in 2021, Chinram and Nakkhasen [6], in 2022, and Gaketem [7], in 2022, introduced the notions of almost subsemigroups of semigroups, almost bi-quasi-interior ideals of semigroups, and almost bi-interior ideals of semigroups, respectively. Furthermore, researchers have studied various types of fuzzifications of almost ideals in [3, 5-9].

Molodtsov [10], in 1999, introduced the concept of soft sets as a mapping from the parameter set E to the power set of U to model uncertainty, which has since garnered significant attention from researchers across various fields. Soft set operations, the foundational concept of the theory, have been extensively studied in [11-26]. Çağman and Enginoğlu modified the definition of soft sets and their operations in [27], while Çağman et al. [28] introduced the notion of soft intersection groups, inspiring the development of several soft algebraic structures.

The application of soft sets in semigroups led to the concept of soft intersection substructures of semigroups. Sezer et al. introduced and studied soft intersection subsemigroups, left (right/two-sided) ideals, (generalized) bi-ideals, interior ideals, and quasi-ideals of semigroups

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in [29,30]. Sezgin and Orbay [31] characterized various types of semigroups, such as semisimple, duo, right (left) zero, and right (left) simple semigroups, as well as semilattices of left (right) simple semigroups and semilattices of groups, in terms of soft intersection substructures of semigroups. Recently, Rao introduced several new types of ideals of semigroups, including bi-interior ideals [32], bi-quasi ideals [33], quasi-interior and weak-interior ideals [34], bi-quasi-interior ideals [35], and Baupradist proposed essential ideals [36]. Soft sets have been studied as a wide range of algebraic structures in [37-48].
In this study, we introduce the notion of soft intersection almost weak-interior ideals, a generalization of nonnull soft intersection weak-interior ideals of semigroups defined in [49]. We demonstrate that the collection of soft intersection almost weak-interior ideals of a semigroup forms a semigroup under the operation of union, for soft sets but not under the operation of intersection for soft sets. We show that the soft intersection almost ideal and soft intersection almost weak-interior ideal coincide with each other when the soft set is idempotent, and also, we obtain that an idempotent soft almost weak-interior ideal is a soft intersection almost subsemigroup. Furthermore, we establish the connection between almost weak-interior ideals and soft intersection weak-interior ideals of a semigroup, particularly regarding minimality, primeness, semiprimeness, and strong primeness.

2. Preliminaries

In this paper, we review several fundamental notions related to semigroups and soft sets.
Definition 2.1. Let U be the universal set, E be the parameter set, $P(U)$ be the power set of U, and $K \subseteq E$. A soft set f_K over U is a set-valued function such that $f_K: E \to P(U)$ such that for all $x \notin K$, $f_K(x) = \emptyset$. A soft set over U can be represented by the set of ordered pairs

$$f_K = \{(x, f_K(x)): x \in E, f_K(x) \in P(U)\}$$

[10,27]. Throughout this paper, the set of all the soft sets over U is designated by $S_E(U)$.

Definition 2.2. Let $f_A \in S_E(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then f_A is called a null soft set and denoted by $\emptyset_E$. If $f_A(x) = U$ for all $x \in E$, then f_A is called an absolute soft set and denoted by U_E [27].

Definition 2.3. Let f_A, $f_B \in S_E(U)$. If for all $x \in E$, $f_A(\mathrm{x}) \subseteq f_B(\mathrm{x})$, then f_A is a soft subset of f_B and denoted by $f_A \,\tilde{\subseteq}\, f_B$. If $f_A(x) = f_B(x)$ for all $x \in E$, then f_A is called soft equal to f_B and denoted by $f_A = f_B$ [27].

Definition 2.4. Let f_A, $f_B \in S_E(U)$. The union of f_A and f_B is the soft set $f_A \,\tilde{\cup}\, f_B$, where $(f_A \,\tilde{\cup}\, f_B)(x) = f_A(x) \cup f_B(x)$ for all $x \in E$. The intersection of f_A and f_B is the soft set $f_A \,\tilde{\cap}\, f_B$, where $(f_A \,\tilde{\cap}\, f_B)(x) = f_A(x) \cap f_B(x)$ for all $x \in E$ [27].

Definition 2.5. For a soft set f_A, the support of f_A is defined by

$$supp(f_A) = \{x \in A: f_A(x) \neq \emptyset\} \text{ [15]}$$

It is obvious that a soft set with an empty support is a null soft set, otherwise the soft set is nonnull.

Note 2.6. If $f_A \,\tilde{\subseteq}\, f_B$, then $supp(f_A) \subseteq supp(f_B)$ [50].

A semigroup S is a nonempty set with an associative binary operation and throughout this paper, S stands for a semigroup and all the soft sets are the elements of $S_S(U)$ unless otherwise specified. A nonempty subset A of S is called a subsemigroup of S if $AA \subseteq A$, and is called an interior ideal of S if $SAS \subseteq A$. A nonempty subset A of S is called a left weak-interior ideal of S if $SAA \subseteq A$, and is called a right weak-interior ideal of S if $AAS \subseteq A$, and is called a weak-interior ideal of S if A is both a left weak-interior ideal of S and a right weak-interior ideal of S [34].

Definition 2.7. A nonempty subset A of S is called an almost left weak-interior ideal of S if for all $s \in S$; $sAA \cap A \neq \emptyset$, and is called an almost right weak-interior ideal of S if for all $s \in S$, $AAs \cap A \neq \emptyset$; and is called an almost weak-interior ideal (briefly almost WI-ideal) of S when A is both an almost left weak-interior ideal of S and an almost right weak-interior ideal of S.
Example 2.8. Let $S = \mathbb{Z}$ and $\emptyset \neq 2\mathbb{Z} \subseteq \mathbb{Z}$. Since $s(2\mathbb{Z})(2\mathbb{Z}) \cap 2\mathbb{Z} \neq \emptyset$ and $(2\mathbb{Z})(2\mathbb{Z})s \cap 2\mathbb{Z} \neq \emptyset$ for all $s \in \mathbb{Z}$, $2\mathbb{Z}$ is an almost weak-interior ideal of S.
An almost left (resp. right) weak-interior ideal A of S is called a minimal almost left (resp. right) weak-interior ideal of S if for any almost left (resp. right) weak-interior ideal B of S if whenever $B \subseteq A$, then $A = B$. An almost left (resp. right) weak-interior ideal P of S is called a prime almost left (resp. right) weak-interior ideal if for any almost left (resp. right) weak-interior ideals A and B of S such that $AB \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$. An almost left (resp. right) weak-interior ideal P of S is called a semiprime almost left (resp. right) weak-interior ideal if for any almost left (resp. right) weak-interior ideal A of S such that $AA \subseteq P$ implies that $A \subseteq P$. An almost left (resp. right) weak-interior ideal P of S is called a strongly prime almost left (resp. right) weak-interior ideal if for any almost left (resp. right) weak-interior ideals A and B of S such that $AB \cap BA \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.

Definition 2.9. Let f_S and g_S be soft sets over the common universe U. Then, soft intersection product $f_S \circ g_S$ is defined by [29]

$$(f_S \circ g_S)(x) = \begin{cases} \bigcup\limits_{x=yz} \{f_S(y) \cap g_S(z)\}, & if\ \exists y, z \in S\ such\ that\ x = yz \\ \emptyset, & otherwise \end{cases}$$

Theorem 2.10. Let $f_S, g_S, h_S \in S_S(U)$. Then,

i) $(f_S \circ g_S) \circ h_S = f_S \circ (g_S \circ h_S)$.
ii) $f_S \circ g_S \neq g_S \circ f_S$, *generally.*
iii) $f_S \circ (g_S \tilde{\cup} h_S) = (f_S \circ g_S) \tilde{\cup} (f_S \circ h_S)$ and $(f_S \tilde{\cup} g_S) \circ h_S = (f_S \circ h_S) \tilde{\cup} (g_S \circ h_S)$.
iv) $f_S \circ (g_S \tilde{\cap} h_S) = (f_S \circ g_S) \tilde{\cap} (f_S \circ h_S)$ and $(f_S \tilde{\cap} g_S) \circ h_S = (f_S \circ h_S) \tilde{\cap} (g_S \circ h_S)$.
v) If $f_S \tilde{\subseteq} g_S$, then $f_S \circ h_S \tilde{\subseteq} g_S \circ h_S$ and $h_S \circ f_S \tilde{\subseteq} h_S \circ g_S$.
vi) If $t_S, k_S \in S_S(U)$ such that $t_S \tilde{\subseteq} f_S$ and $k_S \tilde{\subseteq} g_S$, then $t_S \circ k_S \tilde{\subseteq} f_S \circ g_S$ [29].

Lemma 2.11. Let f_S and g_S be soft sets over U. Then, $f_S \circ g_S = \emptyset_S \Leftrightarrow f_S = \emptyset_S$ or $g_S = \emptyset_S$ [51].

Definition 2.12. Let A be a subset of S. We denote by S_A the soft characteristic function of A and define as

$$S_A(x) = \begin{cases} U, & if\ x \in A \\ \emptyset, & if\ x \in S \backslash A \end{cases}$$

The soft characteristic function of A is a soft set over U, that is, $S_A: S \longrightarrow P(U)$ [29].

Corollary 2.13. $supp(S_A) = A$ [50].

Theorem 2.14. Let X and Y be nonempty subsets of S. Then, the following properties hold [29, 50]:

i) $X \subseteq Y$ if and only if $S_X \tilde{\subseteq} S_Y$
ii) $S_X \tilde{\cap} S_Y = S_{X \cap Y}$ and $S_X \tilde{\cup} S_Y = S_{X \cup Y}$
iii) $S_X \circ S_Y = S_{XY}$

Definition 2.15. Let x be an element in S. We denote by S_x the soft characteristic function of x and define as

$$S_x(y) = \begin{cases} U, & if \;\; y = x \\ \emptyset, & if \;\; y \neq x \end{cases}$$

The soft characteristic function of x is a soft set over U, that is, $S_x\colon S \longrightarrow P(U)$ [51].

Corollary 2.16. Let $x \in S$, f_S and S_x be soft sets over U. Then,

$$S_x \circ f_S \circ f_S = \emptyset_S \;\Leftrightarrow\; f_S = \emptyset_S \;\; (f_S \circ f_S \circ S_x = \emptyset_S \Leftrightarrow f_S = \emptyset_S\,).$$

Proof: By Lemma 2.11, $S_x \circ f_S \circ f_S = \emptyset_S \Leftrightarrow f_S \circ f_S = \emptyset_S$ or $S_x = \emptyset_S$. By Definition 2.15, $S_x \neq \emptyset_S$ and so the rest of the proof is obvious by Lemma 2.11.

Definition 2.17. A soft set f_S over U is called a soft intersection left (resp. right) weak-interior ideal of S over U if $f_S(xyz) \supseteq f_S(y) \cap f_S(z)$ $(f_S(xyz) \supseteq f_S(x) \cap f_S(y))$ for all $x, y, z \in S$. A soft set f_S over U is called a soft intersection weak-interior ideal of S if it is both a soft intersection left weak-interior ideal of S and a soft intersection right weak-interior ideal of S over U [49].
It is easy to see that if $f_S(x) = U$ for all $x \in S$, then f_S is a (left/right) weak-interior ideal of S. We denote such a kind of soft intersection (left/right) weak-interior ideal by $\widetilde{\mathbb{S}}$. It is obvious that $\widetilde{\mathbb{S}} = S_S$, that is, $\widetilde{\mathbb{S}}(x) = U$ for all $x \in S$.
Theorem 2.18. Let f_S be a soft set over U. Then, f_S is a soft intersection left (resp. right) weak-interior ideal of S if and only if $\widetilde{\mathbb{S}} \circ f_S \circ f_S \tilde{\subseteq} f_S$ $(f_S \circ f_S \circ \widetilde{\mathbb{S}} \tilde{\subseteq} f_S)$. f_S is a soft intersection weak-interior ideal of S if and only if $\widetilde{\mathbb{S}} \circ f_S \circ f_S \tilde{\subseteq} f_S$ and $f_S \circ f_S \circ \widetilde{\mathbb{S}} \tilde{\subseteq} f_S$ [49].

Definition 2.19. Let f_S be a soft set over U. Then, f_S is called a soft intersection almost subsemigroup of S if $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ [50]; and is called a soft intersection almost left (right) ideal of S if $(S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ $\big((f_S \circ S_x) \tilde{\cap} f_S \neq \emptyset_S\big)$ for all $x \in S$. f_S is called a soft intersection almost two-sided ideal (or briefly soft intersection almost ideal) of S if f_S is both soft intersections almost left ideal of S and soft intersection almost right ideal of S [51].

Throughout this paper, we prefer to use “SI-left (right) WI-ideal” instead of “soft intersection left (right) weak-interior ideal of S”. We refer to [52] for the discussions of network analysis and graph applications, drawing inspiration from the divisibility of determinants and [53] for complementary soft binary piecewise theta operation.

3. Soft Intersection Almost Weak-Interior Ideals Of Semigroups

Definition 3.1. Let f_S be a soft set over U. f_S is called a soft intersection almost left weak-interior ideal of S if for all $x \in S$,

$$(S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$$

Definition 3.2. Let f_S be a soft set over U. f_S is called a soft intersection almost right weak-interior ideal of S if for all $x \in S$,

$$(f_S \circ f_S \circ S_x) \tilde{\cap} f_S \neq \emptyset_S$$

Definition 3.3. Let f_S be a soft set over U. f_S is called a soft intersection almost weak-interior ideal of S if for all $x \in S$,

$$(S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S \text{ and } (f_S \circ f_S \circ S_x) \tilde{\cap} f_S \neq \emptyset_S$$

Hereafter, for brevity, soft intersection is designated by SI; and weak-interior ideal by WI-ideal. Thus, soft intersection almost left weak-interior ideal, soft intersection almost right weak-interior ideal, and soft intersection almost weak-interior ideal of S are denoted by SI-almost left WI-ideal; SI-almost right WI-ideal; SI-almost WI-ideal**,** respectively. It is obvious that SI-almost WI-ideal is both SI-almost left WI-ideal and SI-almost right WI-ideal of S.

Example 3.4. Let $S = \{\hbar, y, t\}$ be the semigroup with the following Cayley Table.

	$\hbar$	y	t
$\hbar$	t	$\hbar$	$\hbar$
y	$\hbar$	y	t
t	$\hbar$	t	t

Let f_S, h_S, and g_S be soft sets over $U = S_3$ as follows:

$$f_S = \{(\hbar, \{(1), (23)\}), (y, \{(23), (123)\}), (t, \{(12), (13), (132)\})\}$$
$$h_S = \{(\hbar, \{(12), (13)\}), (y, \{(13), (132)\}), (t, \{(1), (23), (123)\})\}$$
$$g_S = \{(\hbar, \{(13), (123), (132)\}), (y, \{(1), (12), (23)\}), (t, \emptyset)\}$$

Here, f_S and h_S are both SI-almost WI-ideals. Let's first show that f_S is an SI-almost left WI-ideal, that is, for all $x \in S$, $(S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$:

Let's start with $(S_\hbar \circ f_S \circ f_S) \tilde{\cap} f_S$:

$$\begin{aligned}
[(S_\hbar \circ f_S \circ f_S) \tilde{\cap} f_S](\hbar) &= (S_\hbar \circ f_S \circ f_S)(\hbar) \cap f_S(\hbar) \\
&= \big[\big(S_\hbar(\hbar) \cap (f_S \circ f_S)(y)\big) \cup \big(S_\hbar(\hbar) \cap (f_S \circ f_S)(t)\big) \\
&\cup \big(S_\hbar(y) \cap (f_S \circ f_S)(\hbar)\big) \cap \big(S_\hbar(t) \cap (f_S \circ f_S)(\hbar)\big)\big] \cap f_S(\hbar) \\
&= [(f_S \circ f_S)(y) \cup (f_S \circ f_S)(t) \cup \emptyset \cup \emptyset] \cap f_S(\hbar) \\
&= \Big[[f_S(y) \cap f_S(y)] \\
&\cup \big[\big(f_S(\hbar) \cap f_S(\hbar)\big) \cup \big(f_S(y) \cap f_S(t)\big) \cup \big(f_S(t) \cap f_S(y)\big) \\
&\cup \big(f_S(t) \cap f_S(t)\big)\big]\Big] \cap f_S(\hbar)
\end{aligned}$$

$$= \left[f_S(y) \cup [f_S(h) \cup f_S(t)]\right] \cap f_S(h)$$
$$= f_S(h)$$

$$[(S_h \circ f_S \circ f_S) \tilde{\cap} f_S](y) = (S_h \circ f_S \circ f_S)(y) \cap f_S(y)$$
$$= [S_h(y) \cap (f_S \circ f_S)(y)] \cap f_S(y)$$
$$= \emptyset$$

$$[(S_h \circ f_S \circ f_S) \tilde{\cap} f_S](t) = (S_h \circ f_S \circ f_S)(t) \cap f_S(t)$$
$$= \left[\left(S_h(h) \cap (f_S \circ f_S)(h)\right) \cup \left(S_h(y) \cap (f_S \circ f_S)(t)\right)\right.$$
$$\left. \cup \left(S_h(t) \cap (f_S \circ f_S)(y)\right) \cup \left(S_h(t) \cap (f_S \circ f_S)(t)\right)\right] \cap f_S(t)$$
$$= [(f_S \circ f_S)(h) \cup \emptyset \cup \emptyset \cup \emptyset] \cap f_S(t)$$
$$= \left[\left(f_S(h) \cap f_S(y)\right) \cup \left(f_S(h) \cap f_S(t)\right) \cup \left(f_S(y) \cap f_S(h)\right)\right.$$
$$\left. \cup \left(f_S(t) \cap f_S(h)\right)\right] \cap f_S(t)$$
$$= \left[\left(f_S(h) \cap f_S(y)\right) \cup \left(f_S(h) \cap f_S(t)\right)\right] \cap f_S(t)$$
$$= \emptyset$$

Consequently,
$$(S_h \circ f_S \circ f_S) \tilde{\cap} f_S = \{(h, \{(1), (23)\}), (y, \emptyset), (t, \emptyset)\} \neq \emptyset_S$$
Similarly,
$$\left(S_y \circ f_S \circ f_S\right) \tilde{\cap} f_S = \{(h, \{(23)\}), (y, \{(23), (123)\}), (t, \{(12), (13), (132)\})\} \neq \emptyset_S$$
$$(S_t \circ f_S \circ f_S) \tilde{\cap} f_S = \{(h, \{(23)\}), (y, \emptyset), (t, \{(12), (13), (132)\})\} \neq \emptyset_S$$

Therefore, for all $x \in S$, $(S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, so f_S is an SI-almost left WI-ideal. Now let's show that f_S is an SI-almost right WI-ideal, that is, for all $x \in S$, $(f_S \circ f_S \circ S_x) \tilde{\cap} f_S \neq \emptyset_S$:

$$(f_S \circ f_S \circ S_h) \tilde{\cap} f_S = \{(h, \{(1), (23)\}), (y, \emptyset), (t, \emptyset)\} \neq \emptyset_S$$
$$\left(f_S \circ f_S \circ S_y\right) \tilde{\cap} f_S = \{(h, \{(23)\}), (y, \{(23), (123)\}), (t, \{(12), (13), (132)\})\} \neq \emptyset_S$$
$$(f_S \circ f_S \circ S_t) \tilde{\cap} f_S = \{(h, \{(23)\}), (y, \emptyset), (t, \{(12), (13), (132)\})\} \neq \emptyset_S$$

Therefore, for all $x \in S$, $(f_S \circ f_S \circ S_x) \tilde{\cap} f_S \neq \emptyset_S$, so f_S is an SI-almost right WI-ideal. Thus f_S is an SI-almost WI-ideal.

Similarly, h_S is an SI-almost left WI-ideal and SI-almost right WI-ideal, thus h_S is an SI-almost WI-ideal. In fact;
$$(S_h \circ h_S \circ h_S) \tilde{\cap} h_S = \{(h, \{(12), (13)\}), (y, \emptyset), (t, \emptyset)\} \neq \emptyset_S$$
$$\left(S_y \circ h_S \circ h_S\right) \tilde{\cap} h_S = \{(h, \{(13)\}), (y, \{(13), (132)\}), (t, \{(1), (23), (123)\})\} \neq \emptyset_S$$
$$(S_t \circ h_S \circ h_S) \tilde{\cap} h_S = \{(h, \{(13)\}), (y, \emptyset), (t, \{(1), (23), (123)\})\} \neq \emptyset_S.$$

Hence, h_S is an SI-almost left WI-ideal. Moreover,

$$(h_S \circ h_S \circ S_h) \tilde{\cap} h_S = \{(h, \{(12), (13)\}), (y, \emptyset), (t, \emptyset)\} \neq \emptyset_S$$
$$\left(h_S \circ h_S \circ S_y\right) \tilde{\cap} h_S = \{(h, \{(13)\}), (y, \{(13), (132)\}), (t, \{(1), (23), (123)\})\} \neq \emptyset_S$$
$$(h_S \circ h_S \circ S_t) \tilde{\cap} h_S = \{(h, \{(13)\}), (y, \emptyset), (t, \{(1), (23), (123)\})\} \neq \emptyset_S.$$

Thus, h_S is an SI-almost right WI-ideal, thus h_S is an SI-almost WI-ideal.

One can also show that g_S is not an SI-almost (left/right) WI-ideal. In fact;

$$[(S_t \circ g_S \circ g_S) \tilde{\cap} g_S](h) = (S_t \circ g_S \circ g_S)(h) \cap g_S(h)$$

$$
\begin{aligned}
&= \big[[S_t(\hbar) \cap (g_S \circ g_S)(y)] \cup [S_t(\hbar) \cap (g_S \circ g_S)(t)] \\
&\cup [S_t(y) \cap (g_S \circ g_S)(\hbar)] \cup [S_t(t) \cap (g_S \circ g_S)(\hbar)]\big] \cap g_S(\hbar) \\
&= [\emptyset \cup \emptyset \cup \emptyset \cup (g_S \circ g_S)(\hbar)] \cap g_S(\hbar) \\
&= \big[\big(g_S(\hbar) \cap g_S(y)\big) \cup \big(g_S(\hbar) \cap g_S(t)\big) \\
&\cup \big(g_S(y) \cap g_S(\hbar)\big) \cup \big(g_S(t) \cap g_S(\hbar)\big)\big] \cap g_S(\hbar) \\
&= \big[[g_S(\hbar) \cap g_S(y)] \cup [g_S(\hbar) \cap g_S(t)]\big] \cap g_S(\hbar) \\
&= \emptyset
\end{aligned}
$$

$$
\begin{aligned}
[(S_t \circ g_S \circ g_S) \tilde{\cap} g_S](y) &= (S_t \circ g_S \circ g_S)(y) \cap g_S(y) \\
&= [S_t(y) \cap (g_S \circ g_S)(y)] \cap g_S(y) \\
&= \emptyset
\end{aligned}
$$

$$[(S_t \circ g_S \circ g_S) \tilde{\cap} g_S](t) = (S_t \circ g_S \circ g_S)(t) \cap g_S(t) = \emptyset$$

for $t \in S$, $(S_t \circ g_S \circ g_S) \tilde{\cap} g_S = \emptyset_S$, thus g_S is not an SI-almost left WI-ideal. Similarly, for $t \in S$,

$$(g_S \circ g_S \circ S_t) \tilde{\cap} g_S = \{(\hbar, \emptyset), (y, \emptyset), (t, \emptyset)\} = \emptyset_S$$

g_S is not an SI-almost right WI-ideal. It is obvious that g_S is not an SI-almost WI-ideal.

From now on, the proofs are given for only SI-almost left WI-ideal, since the proofs for SI-almost right WI-ideal and SI-almost WI-ideal can be shown similarly.

Proposition 3.5. If f_S is an SI-left (resp. right) WI-ideal such that $f_S \neq \emptyset_S$, then f_S is an SI-almost left (resp. right) WI-ideal.

Proof: Let $f_S \neq \emptyset_S$ be an SI-left WI-ideal, then $\tilde{\mathbb{S}} \circ f_S \circ f_S \tilde{\subseteq} f_S$. Since $f_S \neq \emptyset_S$, by Corollary 2.16, it follows that $S_x \circ f_S \circ f_S \neq \emptyset_S$. We need to show that for all $x \in S$,

$$(S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S.$$

Since $S_x \circ f_S \circ f_S \tilde{\subseteq} \tilde{\mathbb{S}} \circ f_S \circ f_S \tilde{\subseteq} f_S$, it follows that $S_x \circ f_S \circ f_S \tilde{\subseteq} f_S$. Thus,

$$(S_x \circ f_S \circ f_S) \tilde{\cap} f_S = S_x \circ f_S \circ f_S \neq \emptyset_S$$

implying that f_S is an SI-almost left WI-ideal.

Here it is obvious that $\emptyset_S$ is an SI-left WI-ideal, as $\tilde{\mathbb{S}} \circ \emptyset_S \circ \emptyset_S \tilde{\subseteq} \emptyset_S$; but it is not SI-almost left WI-ideal, since $(S_x \circ \emptyset_S \circ \emptyset_S) \tilde{\cap} \emptyset_S = \emptyset_S \tilde{\cap} \emptyset_S = \emptyset_S$.

Here note that if f_S is an SI-almost left (resp. right) WI-ideal, then f_S needs not to be an SI left (resp. right) WI-ideal as shown in the following example:

Example 3.6. In Example 3.4, it is shown that f_S and h_S are SI-almost left (resp. right) WI-ideals; however, f_S and h_S are not SI-left (resp. right) WI-ideals. In fact;

$$
\begin{aligned}
\big(\tilde{\mathbb{S}} \circ f_S \circ f_S\big)(t) &= \big[\tilde{\mathbb{S}}(\hbar) \cap (f_S \circ f_S)(\hbar)\big] \cup \big[\tilde{\mathbb{S}}(y) \cap (f_S \circ f_S)(t)\big] \cup \big[\tilde{\mathbb{S}}(t) \cap (f_S \circ f_S)(y)\big] \\
&\cup \big[\tilde{\mathbb{S}}(t) \cap (f_S \circ f_S)(t)\big] \\
&= (f_S \circ f_S)(\hbar) \cup (f_S \circ f_S)(t) \cup (f_S \circ f_S)(y)
\end{aligned}
$$

$$
\begin{aligned}
&= \left[\left(f_S(\hbar) \cap f_S(y)\right) \cup \left(f_S(\hbar) \cap f_S(t)\right) \cup \left(f_S(y) \cap f_S(\hbar)\right)\right. \\
&\quad \left.\cup \left(f_S(t) \cap f_S(\hbar)\right)\right] \\
&\quad \cup \left[\left(f_S(\hbar) \cap f_S(\hbar)\right) \cup \left(f_S(y) \cap f_S(t)\right) \cup \left(f_S(t) \cap f_S(y)\right) \cup \left(f_S(t) \cap f_S(t)\right)\right] \\
&\quad \cup f_S(y) \\
&= f_S(\hbar) \cup f_S(t) \cup f_S(y) \nsubseteq f_S(t)
\end{aligned}
$$

thus, f_S is not an SI-left WI-ideal. Similarly,

$$
\begin{aligned}
\left(f_S \circ f_S \circ \widetilde{\mathbb{S}}\right)(\hbar) &= \left[(f_S \circ f_S)(\hbar) \cap \widetilde{\mathbb{S}}(y)\right] \cup \left[(f_S \circ f_S)(\hbar) \cap \widetilde{\mathbb{S}}(t)\right] \\
&\quad \cup \left[(f_S \circ f_S)(y) \cap \widetilde{\mathbb{S}}(\hbar)\right] \cup \left[(f_S \circ f_S)(t) \cap \widetilde{\mathbb{S}}(\hbar)\right] \\
&= (f_S \circ f_S)(\hbar) \cup (f_S \circ f_S)(y) \cup (f_S \circ f_S)(t) \\
&= f_S(\hbar) \cup f_S(y) \cup f_S(t) \nsubseteq f_S(\hbar)
\end{aligned}
$$

thus, f_S is not an SI-right WI-ideal. It is obvious that f_S is not an SI-WI-ideal. Similarly,

$$\left(\widetilde{\mathbb{S}} \circ h_S \circ h_S\right)(t) = h_S(\hbar) \cup h_S(t) \cup h_S(y) \nsubseteq h_S(t)$$

thus, h_S is not an SI-left WI-ideal. Similarly,

$$\left(h_S \circ h_S \circ \widetilde{\mathbb{S}}\right)(\hbar) = h_S(\hbar) \cup h_S(y) \cup h_S(t) \nsubseteq h_S(\hbar)$$

thus, h_S is not an SI-right WI-ideal. It is clear that h_S is not an SI-WI-ideal.

Proposition 3.7. Let f_S be an idempotent soft set. If f_S is an SI-almost WI-ideal, then f_S is an SI-almost subsemigroup.

Proof: Assume that f_S is an idempotent SI-almost left WI-ideal, then $f_S \circ f_S = f_S$ and $(S_x \circ f_S \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$ for all $x \in S$. We need to show that f_S is an SI-almost subsemigroup, that is $(f_S \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$.

$$
\begin{aligned}
(S_x \circ f_S \circ f_S) \widetilde{\cap} f_S &= [(S_x \circ f_S \circ f_S) \widetilde{\cap} f_S] \widetilde{\cap} f_S \\
&= [(S_x \circ f_S \circ f_S) \widetilde{\cap} (f_S \circ f_S)] \widetilde{\cap} f_S \\
&\widetilde{\subseteq} (f_S \circ f_S) \widetilde{\cap} f_S
\end{aligned}
$$

Since $(S_x \circ f_S \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(f_S \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$. Thus, f_S is an SI-almost subsemigroup.

Theorem 3.8. Let f_S be an idempotent soft set. Then, f_S is an SI-almost ideal if and only if f_S is an SI-almost WI-ideal.

Proof: Assume that f_S is an idempotent soft set over U. Then, $f_S \circ f_S = f_S$. Since,

$$\emptyset_S \neq (S_x \circ f_S) \widetilde{\cap} f_S = (S_x \circ f_S \circ f_S) \widetilde{\cap} f_S \text{ and } \emptyset_S \neq (f_S \circ S_x) \widetilde{\cap} f_S = (f_S \circ f_S \circ S_x) \widetilde{\cap} f_S$$

the rest of the proof is obvious.

Theorem 3.9. Let $f_S \widetilde{\subseteq} h_S$. If f_S is an SI-almost left (resp. right) WI-ideal, then h_S is an SI-almost left (resp. right) WI-ideal.

Proof: Assume that f_S is an SI-almost left WI-ideal. Hence, for all $x \in S$, $(S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$. We need to show that $(S_x \circ h_S \circ h_S) \tilde{\cap} h_S \neq \emptyset_S$. In fact,

$$(S_x \circ f_S \circ f_S) \tilde{\cap} f_S \tilde{\subseteq} (S_x \circ h_S \circ h_S) \tilde{\cap} h_S.$$

Since $(S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(S_x \circ h_S \circ h_S) \tilde{\cap} h_S \neq \emptyset_S$. This completes the proof.

Theorem 3.10. Let f_S and h_S be SI-almost left (resp. right) WI-ideals. Then, $f_S \tilde{\cup} h_S$ is an SI-almost left (resp. right) WI-ideal.

Proof: Since f_S is an SI-almost left WI-ideal by assumption and $f_S \tilde{\subseteq} f_S \tilde{\cup} h_S$, $f_S \tilde{\cup} h_S$ is an SI-almost left WI-ideal by Theorem 3.9.

Corollary 3.11. The finite union of SI-almost left (resp. right) WI-ideals is an SI-almost left (resp. right) WI-ideal.

Corollary 3.12. Let f_S or h_S be SI-almost left (resp. right) WI-ideal. Then, $f_S \tilde{\cup} h_S$ is an SI-almost left (resp. right) WI-ideal.

Here note that if f_S and h_S are SI-almost left (resp. right) WI-ideals, then $f_S \tilde{\cap} h_S$ needs not to be an SI-almost left (resp. right) WI-ideal.

Example 3.13. Consider the SI-almost left (resp. right) WI-ideals f_S and h_S in Example 3.4. Since,

$$f_S \tilde{\cap} h_S = \{(\hbar, \emptyset), (y, \emptyset), (t, \emptyset)\} = \emptyset_S$$

$f_S \tilde{\cap} h_S$ is not an SI-almost left (resp. right) WI-ideals.

Now, we give the relationship between almost WI-ideal and SI-almost WI-ideal. But first of all, we remind the following lemma in order to use it in Theorem 3.15.

Lemma 3.14. Let $x \in S$ and Y be nonempty subset of S. Then, $S_x \circ S_Y = S_{xY}$. If X is a nonempty subset of S and $y \in S$, then it is obvious that $S_X \circ S_y = S_{Xy}$ [51].

Theorem 3.15. Let A be a subset of S. Then, A is an almost left (resp. right) WI-ideal if and only if S_A, the soft characteristic function of A, is an SI-almost left (resp. right) WI-ideal, where $\emptyset \neq A \subseteq S$.

Proof: Assume that $\emptyset \neq A$ is an almost left WI-ideal. Then, $xAA \cap A \neq \emptyset$ for all $x \in S$, and so there exist $t \in S$ such that $t \in xAA \cap A$. Since,

$$\left((S_x \circ S_A \circ S_A) \tilde{\cap} S_A\right)(t) = (S_{xAA} \tilde{\cap} S_A)(t) = (S_{xAA\cap A})(t) = U \neq \emptyset$$

it follows that $(S_x \circ S_A \circ S_A) \tilde{\cap} S_A \neq \emptyset_S$. Thus, S_A is an SI-almost left WI-ideal.
Conversely assume that S_A is an SI-almost left WI-ideal. Hence, we have $(S_x \circ S_A \circ S_A) \tilde{\cap} S_A \neq \emptyset_S$ for all $x \in S$. In order to show that A is an almost left WI-ideal, we should prove that $A \neq \emptyset$ and $xAA \cap A \neq \emptyset$ for all $x \in S$. $A \neq \emptyset$ is obvious from assumption. Now,

$$\begin{aligned} \emptyset_S \neq (S_x \circ S_A \circ S_A) \tilde{\cap} S_A &\Rightarrow \exists n \in S\,;\ \left((S_x \circ S_A \circ S_A) \tilde{\cap} S_A\right)(n) \neq \emptyset \\ &\Rightarrow \exists n \in S\,;\ (S_{xAA} \tilde{\cap} S_A)(n) \neq \emptyset \\ &\Rightarrow \exists n \in S\,;\ (S_{xAA\cap A})(n) \neq \emptyset \\ &\Rightarrow \exists n \in S\,;\ (S_{xAA\cap A})(n) = U \end{aligned}$$

$$\Rightarrow \; n \in xAA \cap A$$

Hence, $xAA \cap A \neq \emptyset$. Consequently, A is an almost left WI-ideal.

Lemma 3.16. Let f_S be a soft set over U. Then, $f_S \tilde{\subseteq} S_{supp(f_S)}$ [50].

Theorem 3.17. If f_S is an SI-almost left (resp. right) WI-ideal, then $supp(f_S)$ is an almost left (resp. right) WI-ideal.

Proof: Assume that f_S is an SI-almost left WI-ideal. Thus, $(S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ for all $x \in S$. In order to show that $supp(f_S)$ is an almost left WI-ideal, by Theorem 3.15, it is enough to show that $S_{supp(f_S)}$ is an SI-almost left WI-ideal. By Lemma 3.16,

$$(S_x \circ f_S \circ f_S) \tilde{\cap} f_S \tilde{\subseteq} \left(S_x \circ S_{supp(f_S)} \circ S_{supp(f_S)}\right) \tilde{\cap} S_{supp(f_S)}$$

and $(S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, it implies that $\left(S_x \circ S_{supp(f_S)} \circ S_{supp(f_S)}\right) \tilde{\cap} S_{supp(f_S)} \neq \emptyset_S$. Consequently, $S_{supp(f_S)}$ is an SI-almost left WI-ideal and by Theorem 3.15, $supp(f_S)$ is an almost left WI-ideal.

Here note that the converse of Theorem 3.17 is not true in general as shown in the following example.

Example 3.18. We know that g_S is not an SI-almost (left/right) WI-ideal in Example 3.4 and it is obvious that $supp(g_S) = \{h, y\}$. Since,

$$[\{h\}supp(g_S)supp(g_S)] \cap supp(g_S) = \{h\}\{h, y\}\{h, y\} \cap \{h, y\} = \{h\} \neq \emptyset$$
$$[\{y\}supp(g_S)supp(g_S)] \cap supp(g_S) = \{y\}\{h, y\}\{h, y\} \cap \{h, y\} = \{h, y\} \neq \emptyset$$
$$[\{t\}supp(g_S)supp(g_S)] \cap supp(g_S) = \{t\}\{h, y\}\{h, y\} \cap \{h, y\} = \{h\} \neq \emptyset.$$

It is seen that $[xsupp(g_S)supp(g_S)] \cap supp(g_S) \neq \emptyset$ for all $x \in S$. That is to say, $supp(g_S)$ is an almost left WI-ideal; although g_S is not an SI-almost left WI-ideal. Similarly,

$$[supp(g_S)supp(g_S)\{h\}] \cap supp(g_S) = \{h, y\}\{h, y\}\{h\} \cap \{h, y\} = \{h\} \neq \emptyset$$
$$[supp(g_S)supp(g_S)\{y\}] \cap supp(g_S) = \{h, y\}\{h, y\}\{y\} \cap \{h, y\} = \{h, y\} \neq \emptyset$$
$$[supp(g_S)supp(g_S)\{t\}] \cap supp(g_S) = \{h, y\}\{h, y\}\{t\} \cap \{h, y\} = \{h\} \neq \emptyset.$$

It is seen that $[supp(g_S)supp(g_S)x] \cap supp(g_S) \neq \emptyset$ for all $x \in S$. That is to say, $supp(g_S)$ is an almost right WI-ideal; although g_S is not an SI-almost right WI-ideal. Consequently, $supp(g_S)$ is an almost WI-ideal; although g_S is not an SI-almost WI-ideal.

Definition 3.19. An SI-almost left (resp. right) WI-ideal f_S is called minimal if any SI-almost left (resp. right) WI-ideal h_S if whenever $h_S \tilde{\subseteq} f_S$, then $supp(h_S) = supp(f_S)$.

Theorem 3.20. A is a minimal almost left (resp. right) WI-ideal if and only if S_A, the soft characteristic function of A, is a minimal SI-almost left (resp. right) WI-ideal, where $\emptyset \neq A \subseteq S$.

Proof: Assume that A is a minimal almost left WI-ideal. Thus, A is an almost left WI-ideal, and so S_A is an SI-almost left WI-ideal by Theorem 3.15. Let f_S be an SI-almost left WI-ideal such that $f_S \tilde{\subseteq} S_A$. By Theorem 3.17, $supp(f_S)$ is an almost left WI-ideal, and by Note 2.6 and Corollary 2.13,

$$supp(f_S) \subseteq supp(S_A) = A.$$

Since A is a minimal almost left WI-ideal, $supp(f_S) = supp(S_A) = A$. Thus, S_A is a minimal SI-almost left WI-ideal by Definition 3.19.
Conversely, let S_A be a minimal SI-almost left WI-ideal. Thus, S_A is an SI-almost left WI-ideal and A is an almost left WI-ideal by Theorem 3.15. Let B be an almost left WI-ideal such that $B \subseteq A$. By Theorem 3.15, S_B is an SI-almost left WI-ideal, and by Theorem 2.14 (i), $S_B \tilde{\subseteq} S_A$. Since S_A is a minimal SI-almost left WI-ideal,

$$B = supp(S_B) = supp(S_A) = A$$

by Corollary 2.13. Thus, A is a minimal almost left WI-ideal.

Definition 3.21. Let f_S, g_S, and h_S be any SI-almost left (resp. right) WI-ideals. If $h_S \circ g_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-prime almost left (resp. right) WI-ideal.

Definition 3.22. Let f_S and h_S be any SI-almost left (resp. right) WI-ideals. If $h_S \circ h_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$,
then f_S is called an SI-semiprime almost left (resp. right) WI-ideal.

Definition 3.23. Let f_S, g_S, and h_S be any SI-almost left (resp. right) WI-ideals. If $(h_S \circ g_S) \tilde{\cap} (g_S \circ h_S) \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-strongly prime almost left (resp. right) WI-ideal.

It is obvious that every SI-strongly prime almost WI-ideal is an SI-prime almost WI-ideal and every SI-prime almost WI-ideal is an SI-semiprime almost WI-ideal.

Theorem 3.24. If S_P, the soft characteristic function of P, is an SI-prime almost left (resp. right) WI-ideal, then P is a prime almost left (resp. right) WI-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-prime almost left WI-ideal. Thus, S_P is an SI-almost left WI-ideal and hence, P is an almost left WI-ideal by Theorem 3.15. Let A and B be almost left WI-ideals such that $AB \subseteq P$. Thus, by Theorem 3.15, S_A and S_B are SI-almost left WI-ideals, and by Theorem 2.14 (i) and (iii),

$$S_A \circ S_B = S_{AB} \tilde{\subseteq} S_P.$$

Since S_P is an SI-prime almost left WI-ideal and $S_A \circ S_B \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$ or $S_B \tilde{\subseteq} S_P$. Therefore, by Theorem 2.14 (i), $A \subseteq P$ or $B \subseteq P$. Consequently, P is a prime almost left WI-ideal.

Theorem 3.25. If S_P, the soft characteristic function of P, is an SI-semiprime left (resp. right) almost WI-ideal, then P is a semiprime almost left (resp. right) WI-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-semiprime almost left WI-ideal. Thus, S_P is an SI-almost left WI-ideal and thus, P is an almost left WI-ideal by Theorem 3.15. Let A be an almost left WI-ideal such that $AA \subseteq P$. Thus, by Theorem 3.15, S_A is an SI-almost left WI-ideal, and by Theorem 2.14 (i) and (iii),

$$S_A \circ S_A = S_{AA} \tilde{\subseteq} S_P.$$

Since S_P is an SI-semiprime almost left WI-ideal and $S_A \circ S_A \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$. Therefore, by Theorem 2.14 (i), $A \subseteq P$. Consequently, P is a semiprime almost left WI-ideal.

Theorem 3.26. If S_P, the soft characteristic function of P, is an SI-strongly prime almost left (resp. right) WI-ideal, then P is a strongly prime almost left (resp. right) WI-ideal, where $\emptyset \neq P \subseteq S$.

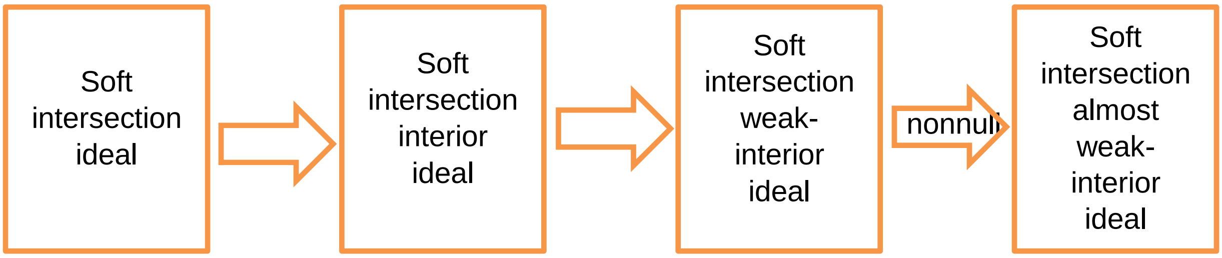
Proof: Assume that S_P is an SI-strongly prime almost left WI-ideal. Thus, S_P is an SI-almost left WI-ideal and thus, P is an almost left WI-ideal by Theorem 3.15. Let A and B be almost left WI-ideals such that $AB \cap BA \subseteq P$. Thus, by Theorem 3.15, S_A and S_B are SI-almost left WI-ideals, and by Theorem 2.14,

$$(S_A \circ S_B) \widetilde{\cap} (S_B \circ S_A) = S_{AB} \widetilde{\cap} S_{BA} = S_{AB\cap BA} \widetilde{\subseteq} S_P$$

Since S_P is an SI-strongly prime almost left WI-ideal and $(S_A \circ S_B) \widetilde{\cap} (S_B \circ S_A) \widetilde{\subseteq} S_P$, it follows that $S_A \widetilde{\subseteq} S_P$ or $S_B \widetilde{\subseteq} S_P$. Thus, by Theorem 2.14 (i), $A \subseteq P$ or $B \subseteq P$. Therefore, P is a strongly prime almost left WI-ideal.

Conclusions

As an extension of a nonnull soft intersection weak-interior ideals of semigroups, we introduced the idea of soft intersection almost weak-interior ideals in this study. We obtained that a semigroup can be constructed under the binary operation of union, but not under the binary operation intersection for soft sets, given the collection of soft intersection almost weak-interior ideals of a semigroup. We also showed that soft intersection almost ideal and soft intersection almost weak-interior ideal coincide with each other when the soft set is idempotent. We also illustrated that an idempotent soft almost weak-interior ideal is a soft intersection almost subsemigroup. Moreover, we showed the relation between soft intersection almost weak-interior ideals of a semigroup, and almost weak-interior ideals of a semigroup by minimality, primeness, semiprimeness, and strongly primeness. In future studies, many types of soft intersection almost ideals, including quasi-ideal, interior ideal, bi-ideal, bi-interior ideal, bi-quasi ideal, quasi-interior ideal, bi-quasi-interior ideal of semigroups and their interrelations can be examined.



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