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DYNAMIC APPROACHES TO MATHEMATICAL PROBLEM MODELING: THE USE OF GEOGEBRA AND WOLFRAM MATHEMATICA FOR ENHANCED SOLUTION

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Abstract

This paper presents the modeling of mathematical tasks which includes their mathematical expression (their mathematical form) and then the usage of mathematical tools to analyze and solve those tasks. To do this, first, we will try to understand the problem (task) which involves the identification of all its important elements and the selection of parameters associated with them. Then we will present mathematical expressions that describe various relations. We will choose the model depending on the nature of the problem and its complexity. In tasks that involve geometry, we will use the GeoGebra application. With this application, we have the opportunity to visualize, explore, and analyze the given model interactively. While, for models that involve analysis and algebra, we will use Wolfram Mathematica, which includes Wolfram Language as its programming language. After using these applications for different kinds of tasks, we will analyze and compare their solution by also solving them in other forms (classical forms for solving mathematical tasks). The purpose of using different applications is to improve the process of mathematical modeling, providing an interactive way to solve tasks, and often allowing faster and more advanced methods that lead to our task solution.

Keywords: Mathematical modeling, Problem-solving, GeoGebra, Wolfram Mathematica, Interactive visualization, Application

1. Introduction

Solving mathematical problems (tasks) nowadays has become easier as technology keeps developing. Other than the classical forms of solving those problems, there are plenty of applications that are created, to help and make this process more attractive and also more concrete, which makes it easier to understand because, through them, there is the possibility that the problem (task) can be seen (presented graphically), as well as the solution. If we had to classify the mathematical tasks (problems) into two groups, we would classify them as geometric tasks and algebraic tasks. For the geometric tasks, we will use the GeoGebra application which gives us the possibility to visualize, explore, and analyze the problem interactively. While, for tasks that involve algebra and also analysis, we will use Wolfram Mathematica, which includes Wolfram Language as its programming language.

2. Material and Methods

While working on the solution of mathematical tasks, despite the usage of classical methods, there was a need, for those problems, for the reason of being well understood, to be visualized and shown more concretely. Knowing that, with chalk and a board, it is very hard to present 3D models, so the usage of the GeoGebra application helps to present these models, and also work on them (by manipulating the parameters). By using Wolfram Language and knowing how to manipulate with programming, the Wolfram application takes us a step closer to technology, which saves time and draws the listener's attention.

3. Results and Discussion

We are going to use Wolfram mathematica to find the solution of congruence tasks, which means, finding the remainder, when k is divided by n ($Mod[k, n]$) (Koshy, 2002). If we have to find the remainder of 293 ($k = 293$) when divided by 12 ($n = 12$), we will have:

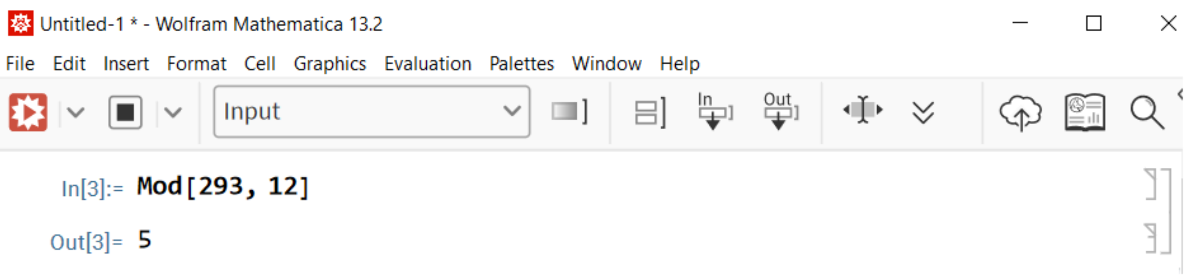


Figure 1. The remainder of 293 when divided by 12

The remainder of (-973) when divided by 11 is:

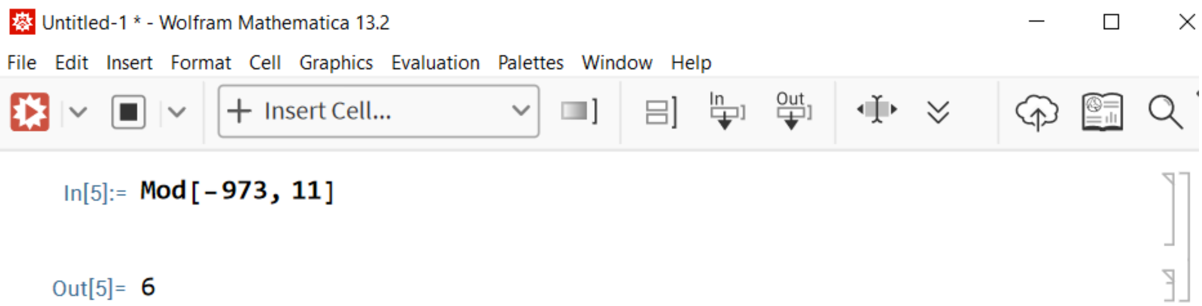


Figure 2. The remainder of (-973) when divided by 11

The remainder of 293^{275} when divided by 48 is ($Mod[k, n, 1]$) (Landau, 2021):

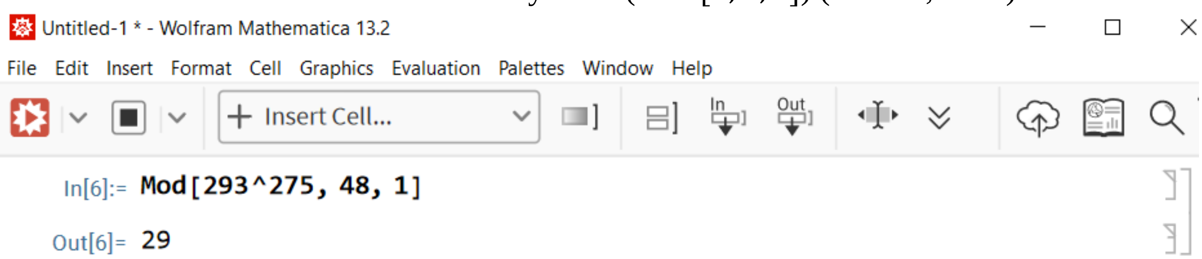
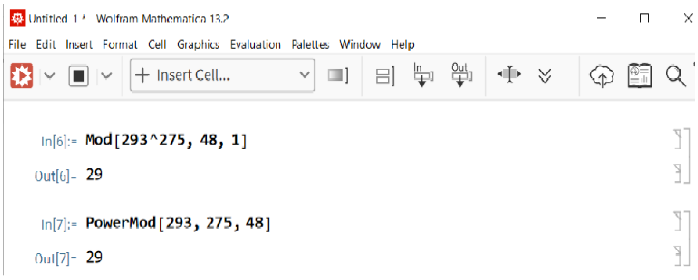


Figure 3. The remainder of 293^{275} when divided by 48

The remainder of 293^{275} when divided by 48 is also ($PowerMod[a, b, n]$, when a^b is divided by n):



$$293^{275} \equiv x(mod48)$$
$$293 \equiv 5(mod28)$$
$$293^2 \equiv 25(mod48)$$
$$293^3 \equiv 29(mod48)$$
$$293^4 \equiv 1(mod48)/^{68}$$
$$293^{4\cdot 68} \equiv 1^{68}(mod48) \Rightarrow 293^{272} \equiv 1(mod48)/\cdot 293^3$$
$$293^{272} \cdot 293^3 \equiv 1 \cdot 29(mod48) \Rightarrow 293^{272+3} \equiv 29(mod48)$$
$$293^{275} \equiv 29(mod48)$$

Figure 4. The remainder of 293^{275} when divided by 48 solved by Wolfram Mathematica and its classical form of solving it

The solution of the given linear congruence $14x \equiv 15(mod45)$ is $(Solve[ax == b, x, Modulus -> n])$ (Dudley, 2012):

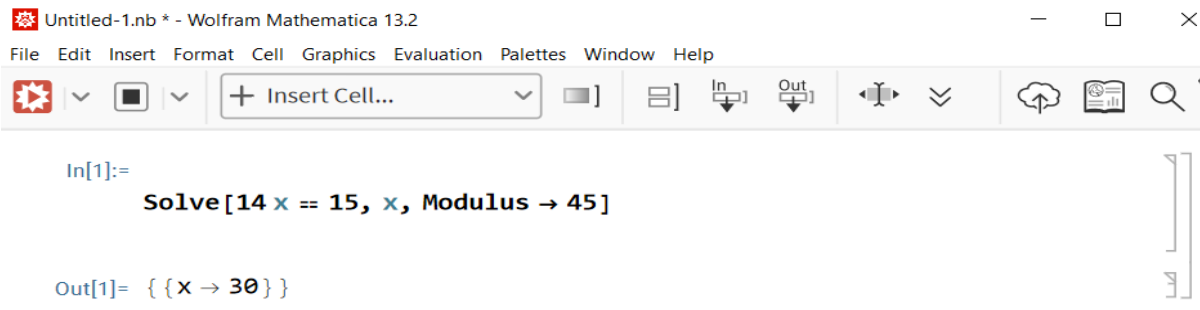
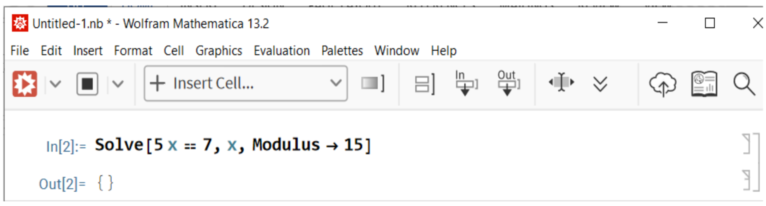


Figure 5. The solution of $14x \equiv 15(mod45)$

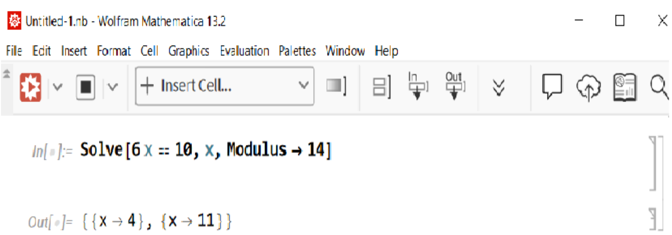
The linear congruence given as $5x \equiv 7(mod15)$ has no solution (Nathanson, 2000), which can be seen down below:



$$5x \equiv 7(mod15)$$
$$\gcd(5,15) = 5 \; but \; 5 \nmid 7 \; our \; equation \; has \; no \; solution.$$

Figure 6. The solution of $5x \equiv 7(mod15)$ solved by Wolfram Mathematica and its classical form of solving it

The linear congruence given as $6x \equiv 10(mod14)$ has more than one solution, which can be seen down below:



$$6x \equiv 10(mod14)$$
$$\gcd(6,14) = 2 \rightarrow two \; possible \; solutions$$
$$6x \equiv 10(mod14)/:2$$
$$\left.\begin{matrix} 3x \equiv 5(mod7) \\ 0 \equiv 7(mod7) \end{matrix}\right\} + \Rightarrow 3x \equiv 5 + 7(mod7) \Rightarrow 3x \equiv 12(mod7)$$
$$3x \equiv 12(mod7) \Rightarrow x \equiv 4(mod7) \Rightarrow x_1 = 4$$
$$x_2 = 4 + 7 = 11 \Rightarrow x_2 = 11$$

Figure 7. The solution of $6x \equiv 10(mod14)$ solved by Wolfram Mathematica and its classical form of solving it

The solution of $4x^3 - 7x^2 + 10 \equiv 0(mod11)$ is (Tattersall, 1999):

Figure 8. Two solutions of $4x^3 - 7x^2 + 10 \equiv 0(mod11)$ solved by Wolfram Mathematica and its classical form of solving it

The congruence $x^3 - 2x - 2 \equiv 0(mod5)$ has no solution (Eynden, 2006), as can be seen down below:

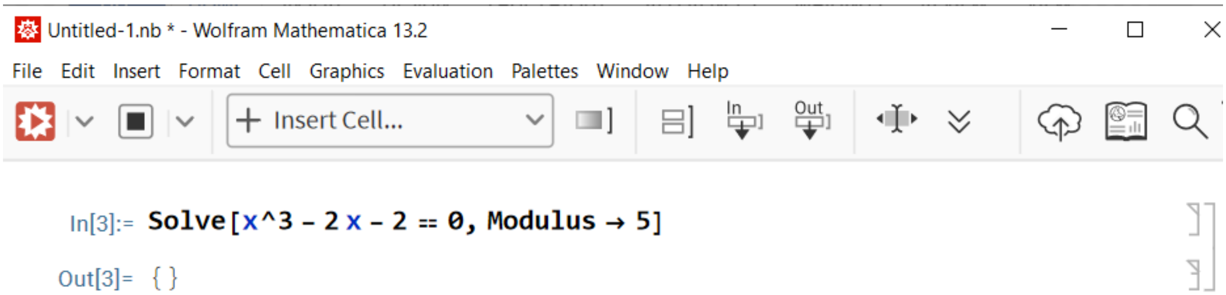


Figure 9. The solution of $x^3 - 2x - 2 \equiv 0(mod5)$

By using the Chinese Remainder Theorem (Rosen, 2011), the solution of our system

$$\begin{cases} x \equiv 3(mod4) \\ x \equiv 1(mod5) \\ x \equiv 2(mod3) \end{cases} \text{ is:}$$

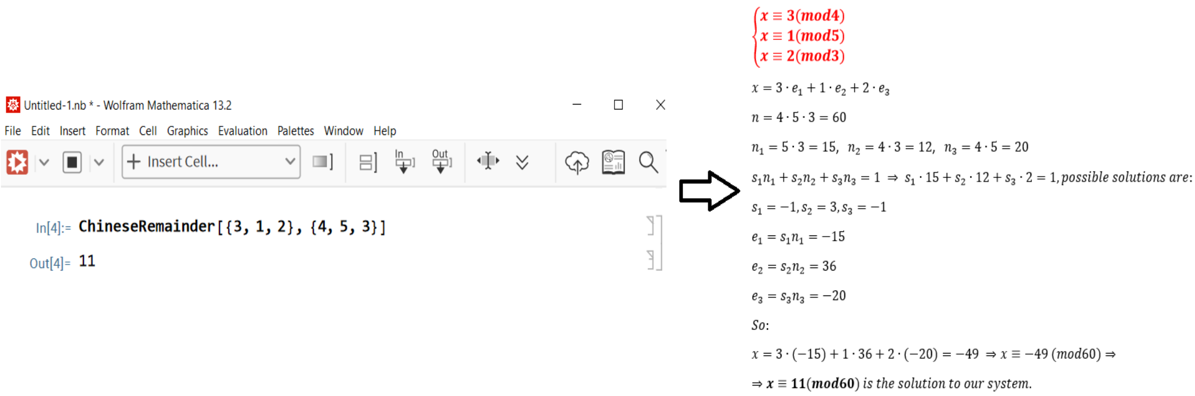


Figure 10. Using the Chinese Remainder Theorem to solve our system with Wolfram Mathematica and in a classical form

The number of integers between 1 and 165, which are coprime to 165 is (Tattersall, 1999):

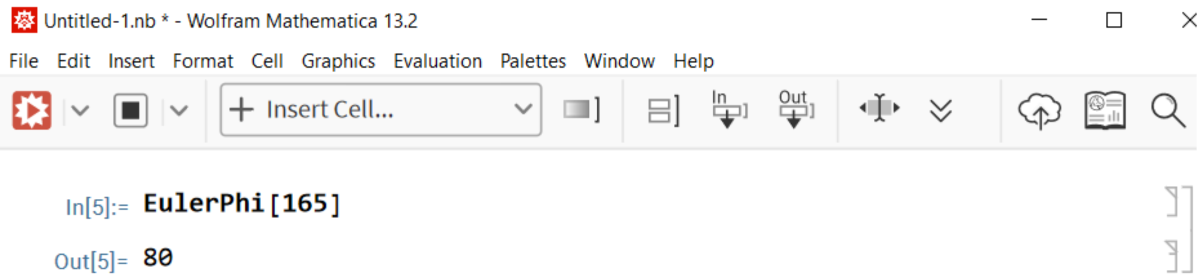


Figure 11. Phi-function of 165

The number of integers between 1 and 10!, which are coprime to 10! Is (Jones, (1998):

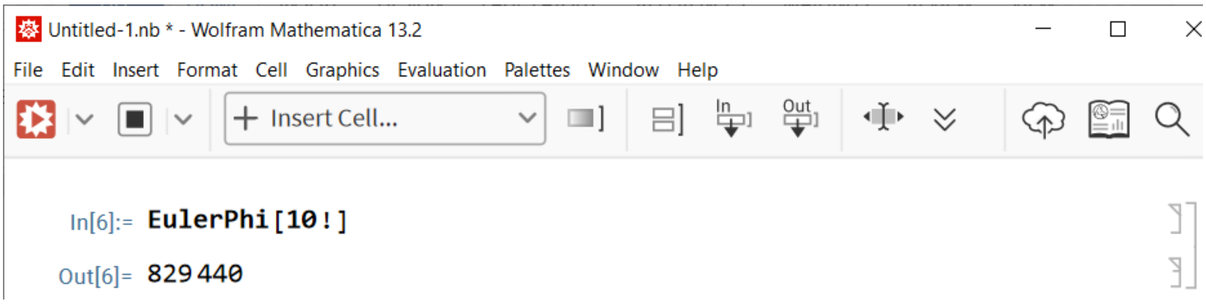


Figure 12. Phi-function of 10!

With GeoGebra we are going to construct the incircle and circumcircle of a square (Majerek, 2014), as can be seen below:

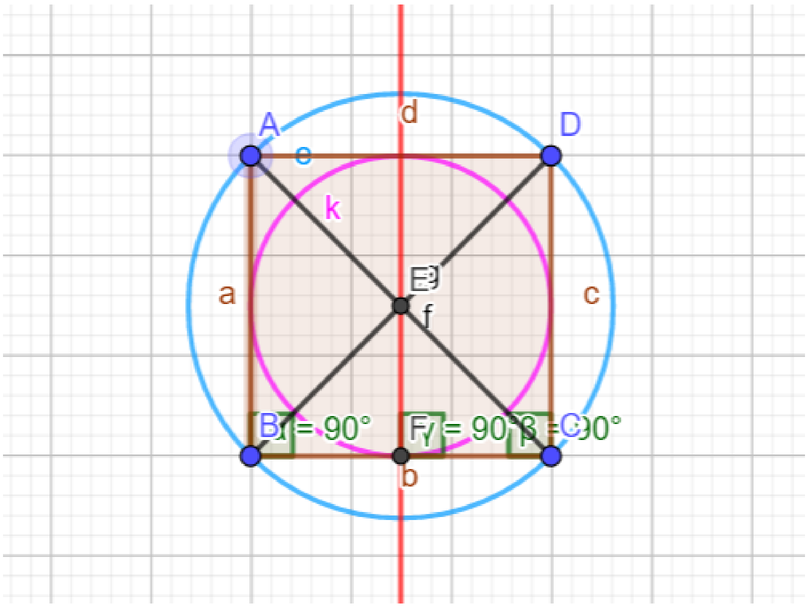


Figure 13. Construction of the incircle and circumcircle of a square

Construction of incircle requires:

1. Drawing the angle bisectors of $\measuredangle B$ and $\measuredangle C$. They intersect at point E.
2. Drawing EF perpendicular to BC.
3. With E as its center and EF as its radius, we construct the incircle of our square.

Construction of circumcircle requires:

1. Drawing the diagonals AC and BD.
2. Their intersection, point E, is the center of our circle with EB radius.

4. Conclusions

From what was said above we can conclude that by using Wolfram Mathematica we are a step closer to technology and programming. With Wolfram Mathematica solving mathematical tasks becomes easier and way faster. We can draw the listener's attention because it attracts everyone to try solving different tasks, rather than just writing or listening to the presenter. With GeoGebra geometric tasks are easier to analyze, because they can be seen and its easier to manipulate with parameters. It gives you the chance to use tools just like in real life, but in a more precise way, which leads to a more precise solution.

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