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Review article

FUNCTIONAL DEPENDENCE BETWEEN CYCLIST SPEED AND VEHICLE SPEED IN HEAD-ON COLLISIONS

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Abstract

Summary: This paper focuses on analyzing the functional dependence between the speed of motor vehicles and the speed of cyclists during mutual collisions, with the goal of determining their influence on the cyclist's body throw distance. The research is based on 810 simulated traffic accidents conducted using the PC Crash 9.0 software package. The simulations involve collisions between ten different types of motor vehicles, each traveling at nine different speeds (25 km/h, 30 km/h, 35 km/h, 40 km/h, 45 km/h, 50 km/h, 55 km/h, and 60 km/h), and cyclists moving at three speeds (15 km/h, 20 km/h, and 25 km/h).

To determine the functional dependence, nonlinear regression models were applied. Initially, power regression was examined, followed by polynomial and exponential models to account for stronger nonlinearities. Additionally, Pearson and Spearman correlation tests were used to identify whether the relationship between travel speeds and the cyclist's throw distance is linear or nonlinear. Artificial intelligence was also utilized in the analysis and interpretation of the results, as well as in the creation of some visualizations, in order to enhance the efficiency and accuracy of the research.

Based on the obtained results, the most appropriate mathematical model describing this relationship is a combined model with logarithmic terms, demonstrating a high coefficient of determination. This confirms that the proposed model accurately explains the variability of the data. Further statistical tests validate the statistical significance of the obtained parameters.

Keywords: Traffic Accident Simulation, PC Crash, Application of Mathematical Models

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Introduction

The safety of cyclists represents a significant challenge in modern traffic systems, primarily due to their exposure to direct collisions with motor vehicles. As a vulnerable category of road users, cyclists are frequently involved in severe traffic accidents, the consequences of which stem from a complex interaction of various kinematic and dynamic factors. Among the most critical parameters determining the outcome of such collisions are the speed of the vehicle, the speed of the cyclist, and the angle of impact – where the post-collision body throw distance emerges as a key indicator for event reconstruction.

In reconstructing these types of collisions, it is necessary not only to rely on empirical data but also to analytically understand the mechanisms that influence the motion of the body after impact. Numerous studies suggest that the relationship between initial speeds and throw distance cannot always be described by a simple linear dependency, emphasizing the need for precise and statistically validated mathematical models. It is especially important that such models are derived from real – world data and allow for practical application in traffic forensics and judicial analysis.

This study applies advanced statistical and regression techniques in order to determine a clear functional relationship between key variables: vehicle speed, cyclist speed, and body throw distance. At the same time, through the analysis of real-world scenarios, this research enables the formalization of these relationships in the form of highly accurate mathematical functions. This approach not only enhances the predictive capability regarding collision outcomes but also establishes a more reliable foundation for planning traffic safety improvement measures aimed at protecting vulnerable road users.

1. Research methodology

This study is based on an analytical and simulation-based approach aimed at examining the functional relationship between motor vehicle speed, cyclist speed, and body throw distance in the case of a central collision. As part of this methodological framework, computer simulations were conducted using *PC Crash 9.0* software, which enables high-precision modeling of collisions based on physical principles and validated kinematic algorithms. This software is specifically designed for traffic accident reconstruction and allows for the reproduction of real – world scenarios under standardized conditions, ensuring accurate measurement of output parameters and valid comparison of results.

Through the controlled simulation of various scenarios, the study covers a significant number of combinations of vehicle and cyclist speeds. Vehicle speeds range from 25 to 60 km/h, while cyclist speeds vary from 15 to 25 km/h. For ten different types of vehicles, a total of 810 simulation scenarios were generated, with body throw distance examined as the key output parameter in each case. All simulations were conducted under controlled conditions – on a flat and clean road surface, free from aerodynamic influences and other external factors – in order to isolate the effect of speed as an independent variable as precisely as possible.

The body throw distance is defined as the horizontal distance traveled by the cyclist's body from the moment of impact to the final state of rest. This parameter is determined by various physical factors, but within the scope of this study, it is analyzed under standardized conditions with a focus on the influence of initial velocities. Each simulation is modeled such that the cyclist crosses the roadway perpendicularly, while the vehicle moves in a straight line and impacts the cyclist with its front end. The point of contact, road surface, and impact angle were held constant across all scenarios, enabling a statistically clean analysis of individual variable effects.

2. Analysis and evaluation of regression models for dependency modeling

To model the relationship between the vehicle speed and the body throw distance of the cyclist following a collision, polynomial, exponential, and power regression models were applied. These modeling techniques are commonly used for capturing nonlinear relationships, offering a better fit to empirical data compared to standard linear regression.

The calculations included polynomial models of the 3rd, 4th, 5th, 6th, and 7th degree, as well as power and exponential models, in order to determine which approach provides the most accurate predictions. For each model, an accuracy analysis was conducted using statistical performance metrics such as *SSE* (*Sum of Squared Errors*), *MSE* (*Mean Squared Error*), and *RMSE* (*Root Mean Squared Error*).

All computations and analyses were carried out using the *Python* programming language, employing libraries such as *NumPy* (for matrix operations), *Pandas* (for data processing), *SciPy* (for nonlinear fitting), and *Matplotlib* (for graphical visualization of results). The *Ordinary Least Squares* (*OLS*) method was used to determine the coefficients of the polynomial models via matrix operations on the design matrix *X*.

Model Analysis and Selection of the Optimal Model for Cyclists Traveling at 15 km/h, 20 km/h, and 25 km/h.

Based on the conducted comparative analysis, it was determined that the 6th-degree polynomial regression model most accurately follows the original measurements and replicates the natural variations in the data, thus presenting itself as the most suitable model for prediction.

On the other hand, the exponential model exhibits excessively rapid growth, leading to deviations, while the power model shows certain divergences from the overall trend, reducing its precision. These differences are visually apparent in the corresponding plot.

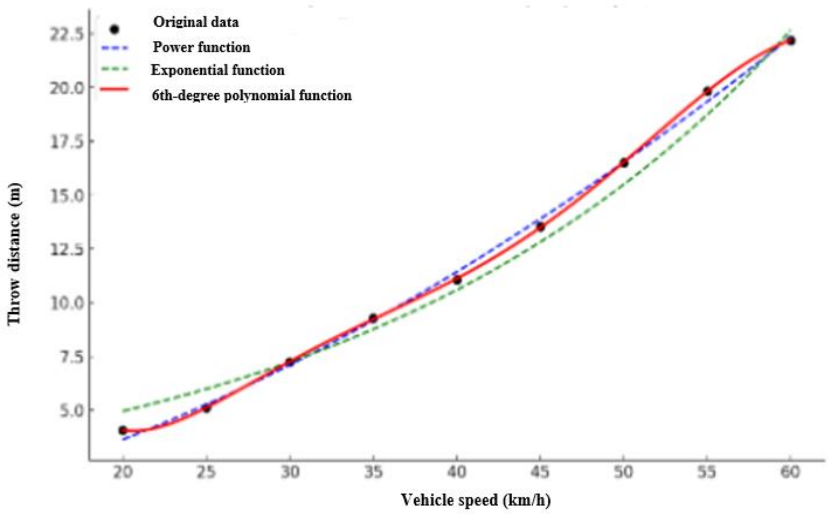

Figure 1. Comparison of Regression Functions for a Cyclist Traveling at 15 km/h

An analogous analysis was conducted for a cyclist traveling at 20 km/h. As in the previous case, the 6th-degree polynomial regression proved to be the most optimal model, with minimal deviation between the predicted and actual data. Furthermore, its coefficient of determination indicates that nearly 100% of the variation in the data can be explained by this model.

The graphical representation confirms this conclusion – the 6th-degree polynomial function most accurately follows the shape of the data, whereas the exponential and power functions exhibit noticeable deviations that affect the model’s overall stability.

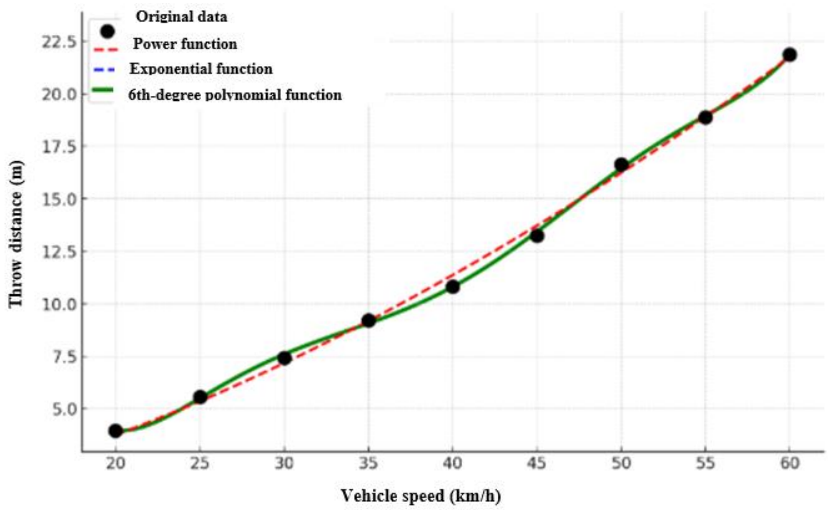

Figure 2. Comparison of Regression Functions for a Cyclist Traveling at 20 km/h

For the final dataset (cyclist traveling at 25 km/h), the results of the comparative analysis likewise demonstrated that the 7th-degree polynomial regression is the most accurate model, precisely capturing the nonlinear variations and enabling stable and reliable predictions (Figure 3).

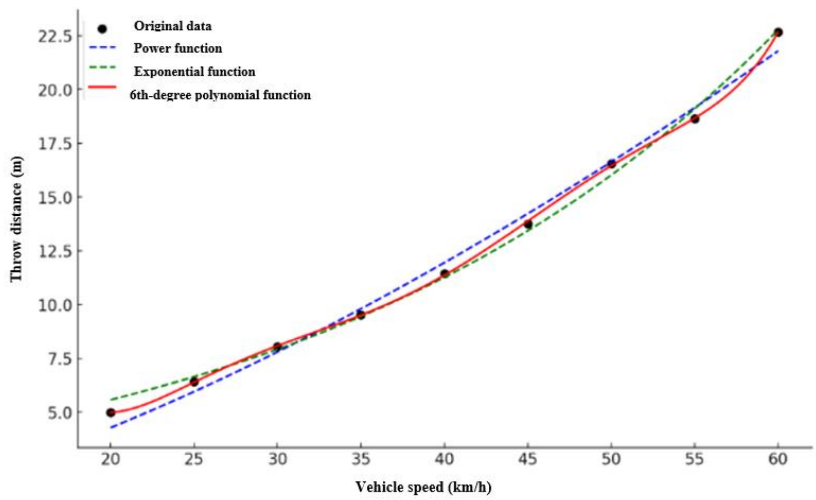

Figure 3. Comparison of Regression Functions for a Cyclist Traveling at 25 km/h

3. Determining the functional relationship between vehicle speed, cyclist speed, and the body throw distance of the cyclist

In this study, a structured and systematic approach was applied to formulate a functional relationship that describes the cyclist's body throw distance as a function of the vehicle speed and cyclist speed. Initially, multiple types of regression models were tested to identify the most appropriate mathematical form. Linear regression demonstrated low accuracy ($R^2 < 0{,}8$), while a second-degree polynomial model provided better, yet still limited, fit to the data.

The best results were achieved using a combined nonlinear model with a logarithmic component, which accurately describes the relationship with a coefficient of determination of $R^2 = 0{,}9975$.

The model was derived through nonlinear regression, the least squares method, and optimization via the *Levenberg–Marquardt* algorithm, implemented in *Python*. The final formula has the following structure:

$$S_{otfv} = a \cdot V_n^m + b \cdot V_v^n + c \cdot V_n \cdot V_v + d \ln(V_n + 1) + e$$

Following the completed analysis and optimization, the resulting parameter values are as follows:

$a = 0,00412$, $m = 2,0722$, $b = 0,0000543$, $n = 3,3210$, $c = -0,00391$, $d = 3,0124$, $e = -6,6468$

By substituting these values into the equation, the final functional form was obtained:

$$S_{otfv} = 0,00412 \cdot V_n^{2,0722} + 0,0000543 \cdot V_v^{3,3210} - 0,00391 \cdot V_n \cdot V_v + 3,0124 \cdot \ln(V_n + 1) - 6,6468$$

The power dependence on the vehicle speed ($V_n^{2,0722}$), that is, the relationship between the throw distance and the vehicle's speed, is nonlinear.

The cubic dependency on cyclist speed ($V_V^{3,3210}$), indicating that the cyclist's speed significantly affects the throw distance.

The interaction term, $-0,00391 \cdot V_n \cdot V_v$ takes into account the effects of the mutual influence of the two speed.

The logarithmic component $3,0124 \cdot \ln(V_v + 1)$ in the model makes it possible to capture and describe the complex nonlinear dynamics that occur at higher speeds. These effects influence the throw distance of the cyclist's body, and the logarithmic term enhances the model, making it more accurate and efficient in its predictions. Essentially, the logarithmic term adjusts the model to better describe the complex interaction between the vehicle speed, the cyclist's speed, and the dynamics of the collision under real-world conditions.

The constant (– 6,6468), adjusts the output values for optimal alignment with the data.

The accuracy analysis of the proposed mathematical model reveals exceptionally high predictive performance, with an R^2 value of 0,9975, indicating that the model explains 99,75% of the variation in throw distance. This result clearly confirms the model's reliability and applicability in real-world cyclist–vehicle collision scenarios.

The *F* – test results indicate that the model as a whole is statistically significant, with an extremely low p-value. Furthermore, *t* – tests conducted on individual parameters reveal that most, particularly vehicle speed (with an exponent of 0,0722), have a highly significant influence ($p < 0,01$), confirming their key role in accurate modeling.

Correlation tests further strengthen the model's credibility. The computed values of the *Pearson* correlation coefficient ($r = 0,9987$) and *Spearman* rank correlation coefficient ($\rho = 0,9939$) indicate an extremely strong linear and ordinal association between the predicted and actual values. The very low *p* – values in both cases confirm that these correlations are statistically significant.

Despite the presence of high multicollinearity (with *VIF* values exceeding 10 for some parameters), the original model remains superior in terms of accuracy and stability. Alternative models such as *Ridge*, *Lasso*, and *PCA* were evaluated but yielded only marginal improvements or even slight decreases in accuracy (R^2 ranging from 0,9964 to 0,9966), suggesting that the elimination of certain variables negatively affects the model's precision.

Additional confirmation of the model's accuracy was obtained through error analysis, showing:

MAE = 0,246 m,
RMSE = 0,30 m,
Mean Absolute Percentage Error = 1,76%.

Even in the most extreme cases, deviations did not exceed the limit of 5,22%, confirming the model's robustness. The – test ($p = 0,884$) revealed no statistically significant differences

between predicted and actual values, while the *Durbin – Watson test* (*DW* = 2,65) confirmed the absence of autocorrelation in the residuals.
The *Kolmogorov – Smirnov* and *Chi – squared tests* indicated that the distribution of predicted values does not significantly differ from the actual ones, further validating the model's quality. The *ANOVA* test, with a very low *p* – value, confirms the model's strong explanatory power.
In summary, the proposed combined nonlinear model is confirmed to be highly accurate, statistically valid, and practically applicable for reconstructing collisions involving cyclists. The results from extensive testing and analysis affirm its effectiveness, stability, and utility in real – world application – without the need for significant modification.

4. Conclusions

Within the scope of this study, a functional relationship has been successfully formulated that enables high-precision prediction of the cyclist's body throw distance following a collision with a motor vehicle, taking into account the vehicle speed and cyclist speed as the primary influencing factors. A systematic and methodologically consistent approach was applied, encompassing data collection through simulations, selection and testing of regression models, parameter calculation, and accuracy validation using multiple statistical methods.
The model that proved to be the most appropriate is a combined nonlinear function incorporating both power and logarithmic elements, allowing for a realistic representation of the complex dynamics involved in a collision. The final formula integrates vehicle speed, cyclist speed, and their interaction within a unified analytical framework, yielding a high coefficient of determination. Additional statistical tests confirm that the model demonstrates high accuracy, statistical significance, and stability.
Although the presence of multicollinearity was identified in some of the model terms, alternative techniques such as *Ridge*, *Lasso*, and *PCA* did not lead to an improvement in predictive accuracy. On the contrary, they reduced the model's predictive quality, indicating that all original variables play an essential role in explaining the dependency.
Further error analysis revealed minimal deviations, even under extreme input values, with a mean absolute percentage error below 2%. Tests for autocorrelation confirmed the model's stability and the absence of systematic bias. Correlation analyses showed near-perfect agreement between observed and predicted values.
In conclusion, the proposed model is verified as stable, statistically valid, and practically applicable in the reconstruction of cyclist – related traffic accidents. It can serve as a valuable tool for expert analysis, engineering assessment, and judicial applications, enabling objective determination of collision dynamics and the identification of key parameters based on the measured throw distance. The model has been fully validated and is ready for integration into practical traffic safety analysis systems.

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