PUPILS' ATTITUDES TOWARD THE NATURE OF MATHEMATICS: FROM FORMULAS AND CALCULATION TO REASONING AND INQUIRY

Mirlinda SHAQIRI[1], Makfirete AMETI[1], Afërdita ILAZI-HOXHA[1], Besa HAVZIU[1]

[1]*Faculty of Pedagogy, University of Tetovo, Ilinden n.n., 1200 Tetovo, Republic of North Macedonia*
*Corresponding author e-mail: mirlinda.shaqiri@unite.edu.mk

Abstact

Mathematics is often perceived as a cold science, filled with numbers, symbols and unchanging rules. In an education system that aims to develop critical and creative thinking, it becomes important to understand the perceptions that children have about the nature of mathematics. This study aims to identify the perceptions of fifth grade pupils about mathematics, examining whether they consider it as a field based primarily on rules, or as a process that fosters critical thinking, discovery and creativity. The research included 188 subjects, fifth grade elementary school pupils in two cities of the Republic of North Macedonia.
The data collected were processed using the SPSS Statistics 25 software, with gender-based comparisons employed to assess differences in perceptions between boys and girls studying in the Albanian and Macedonian languages. The results indicated that pupils studying in Albanian primarily perceive mathematics as a field built upon specific rules, whereas pupils studying in Macedonian view it as an investigative process involving critical thinking, discovery, and creativity.
The perception among pupils that mathematics is merely a set of rules and procedures signals a need to redefine how we teach and understand the subject. Implementing instructional practices that foster 21st-century competencies and connect to real-life contexts can help develop a deeper and richer understanding of mathematics. This redefinition is essential for preparing pupils for future challenges and helping them become informed, engaged citizens in a rapidly changing world.

Keywords: *attitudes; mathematics education; comparison; gender; primary school.*

Introduction

Mathematics is one of the most important areas of the school curriculum and one of the strongest foundations for the development of logical thinking and problem-solving. It is not merely a system of rules and algorithms, but also a way to understand the world around us, to engage in reasoning, and to construct models that describe real-world phenomena (Kilpatrick, Swafford, & Findell, 2001).
In recent decades, the emphasis in mathematics education has shifted increasingly from rote or procedural learning toward conceptual understanding an approach that fosters critical thinking, exploration, and creativity among pupils (Boaler, 2016). Nevertheless, in day-to-day classroom practice, many pupils still perceive mathematics as a rigid subject one based solely on rules and single, definitive solutions which negatively impacts their motivation and engagement (Hiebert & Lefevre, 1986).
Generally accepted definitions of affect, beliefs, attitudes, and emotions are provided in Chapter 7 of the Second Handbook of Research in Mathematics Teaching and Learning (Lester (Ed.), 2007).
Affect is "a disposition or tendency, or an emotion or feeling, associated with an idea or object" (Phillip, 2007, p. 259). It consists of:

- Attitudes – ways of acting, feeling, or thinking that reveal an individual's disposition or opinion;
- Emotions – feelings or states of consciousness, distinct from cognition;
- Beliefs – psychologically accepted understandings, premises, or assertions about the world that are held to be true.

Beliefs are considered lenses that “filter certain complexity of the situation to make it understandable, shaping individuals’ interpretations of events” (Grant, Hiebert & Wearne, 1998, as cited in Phillip, 2007, p. 270). Beliefs influence perception and predispose individuals toward action; they develop gradually, with cultural factors playing a key role in their formation. Certain beliefs (primary) serve as the foundation for other beliefs within a quasi-logical structure. Central beliefs are firmly held, whereas peripheral beliefs are more susceptible to change. Beliefs are organized into clusters that remain relatively isolated from one another. To an observer, belief systems may appear contradictory or inconsistent, as they are specific to the context and the given situation (Phillip, 2007).

In his chapter written for the Second Handbook of Research on Mathematics Teaching (Lester, 2007), Randolph Philipp focused on what he termed "teachers' orientations."

The beliefs that teachers hold regarding mathematics and the learning of mathematics can influence how they teach, and, consequently, can also affect how their pupils learn. The items regarding beliefs about the nature of mathematics explore how respondents conceptualize mathematics as a subject (e.g., mathematics as formal, structural, procedural, or applied). Two scales were developed:

1. **Mathematics as a Set of Rules and Procedures:** Respondents who score high on this scale tend to view mathematics as a collection of procedures to be learned, with strict rules regarding what is correct and what is incorrect.
2. **Mathematics as a Process of Inquiry:** Respondents who score high on this scale view mathematics as a means of answering questions and solving problems. They perceive mathematical procedures as tools for inquiry as means to achieve an end rather than as ends in themselves.

To foster successful and meaningful teaching, it is essential to listen to the voices of the pupils themselves. This educational concept highlights the importance of actively involving children in the learning process by taking into account their perceptions, experiences, and ideas (Lundy, 2007).

In an era of profound changes within the educational system including technology, curricular developments, and a focus on life competencies it is more important than ever to understand how fifth-grade pupils perceive mathematics. Do they view it as a field open to exploration, or as a set of immutable rules? Do they believe that there are various ways to solve a problem, or do they feel constrained by the “only correct way”? Do they perceive a connection between mathematics and their daily lives?

This research aims to explore precisely these questions, drawing on the perceptions of fifth-grade pupils through a questionnaire composed of selected statements that measure attitudes toward the nature of mathematics.

In this research, we focus on confirming the following research hypothesis:

H: There are statistically significant differences between groups of pupils regarding their beliefs about the nature of mathematics.

Methodology

The sample was drawn from the population of fifth-grade pupils, aged 10–11 years. The total sample comprises 188 participants from several primary schools in two cities in the Republic: Tetovo and Skopje.

The focus of this research is the analysis of the system of attitudes, thoughts, and beliefs regarding the nature of mathematical knowledge. In terms of content, this framing of the research focus entails an exploratory examination of the attitudes, thoughts, and beliefs held by pupils at the end of primary education, concentrating on specific aspects related to the nature of mathematics within the system of sciences.

A Likert-type questionnaire was used as an instrument to study pupils' beliefs about the nature of mathematics. For this project, the questionnaire was adapted and translated from an instrument designed as part of the TEDS-M study (Tatto et al., 2012).
The obtained data were processed using the SPSS Statistics 25 software, applying an independent samples t-test.

Results

Pupils' views on the nature of mathematics vary, ranging from those who perceive it as a distant and difficult discipline to those who see it as a creative and powerful tool for understanding the world.
Table 1 presents the aggregated research results, expressed as the percentage of responses supporting statements across various scales.
Following the approach adopted in the TEDS-M studies (Tatto et al., 2012), responses of "agree" or "strongly agree" are considered to represent unqualified endorsement, while responses “strongly disagree”, “disagree”, “slightly disagree” or “slightly agree” are considered to represent non-endorsement or weak endorsement of the statements.
A brief descriptive overview of the content of Table 1:
Regarding beliefs about the nature of mathematics, pupils show the strongest agreement with the statements: "Mathematics is a collection of rules and procedures that prescribe how to solve a problem" (the segment "*Mathematics as a Set of Rules and Procedures*"), and "Mathematical problems can be solved correctly in many ways" (the segment "*Mathematics as a Process of Inquiry*").

Table 1. *Beliefs about mathematics: percentage of various degrees of endorsement*

Statements reflecting	Number of valid responses *N*	Disagree Strongly disagree %	Slightly disagree Slightly agree %	Agree Strongly Agree %
Mathematics as a Set of Rules and Procedures:				
Mathematics is a collection of rules and procedures that prescribe how to solve a problem.	188	.5	15.4	84.0
When solving mathematical tasks, you need to know the correct procedure, else you would be lost.	188	7.4	20.4	71.8
Mathematics involves the remembering & application of definitions, formulas, mathematical facts, & procedures.	188	5.3	29.8	64.9
Mathematics as a Process of Inquiry:				
In mathematics many things can be discovered and tried out by one self.	188	9.6	33.5	56.9
Mathematical problems can be solved correctly in many ways.	188	6.9	31.4	61.7
Mathematics helps solve everyday problems and tasks.	188	11.2	28.2	60.6

Each of the statements in the table above is presented in a pie chart, which helps us better understand the pupils' opinions.

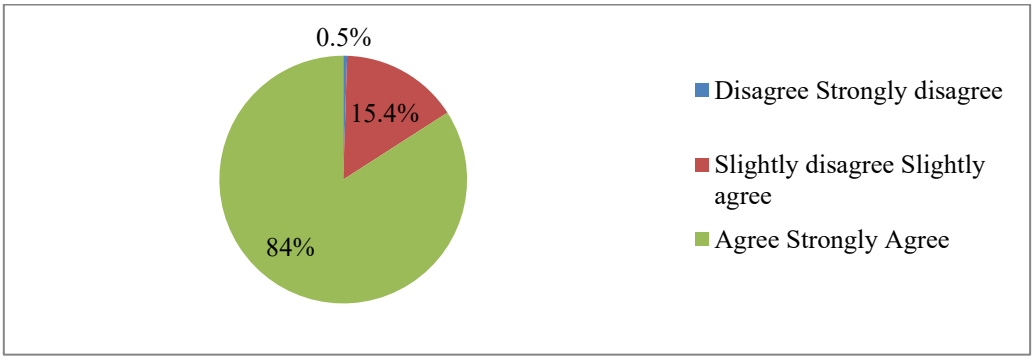


Figure 1. *Graphical representation of the distribution for the statement: Mathematics is a collection of rules and procedures that prescribe how to solve a problem.*

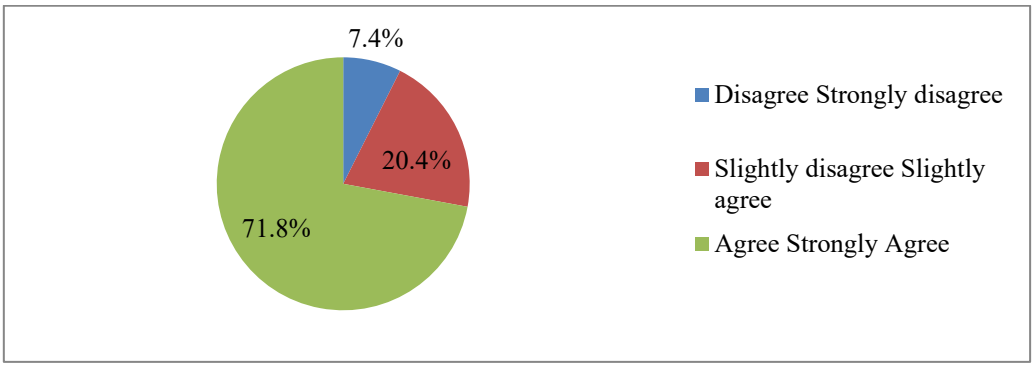


Figure 2. *Graphical representation of the distribution for the statement: When solving mathematical tasks, you need to know the correct procedure, else you would be lost.*

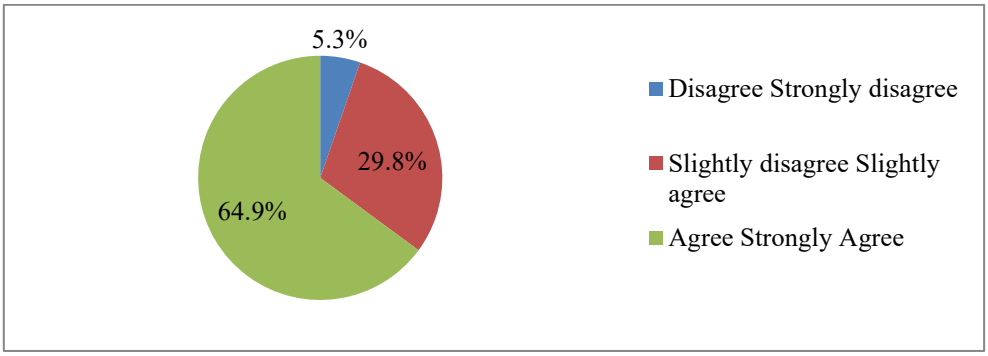


Figure 3. *Graphical representation of the distribution for the statement: Mathematics involves the remembering & application of definitions, formulas, mathematical facts, & procedures.*

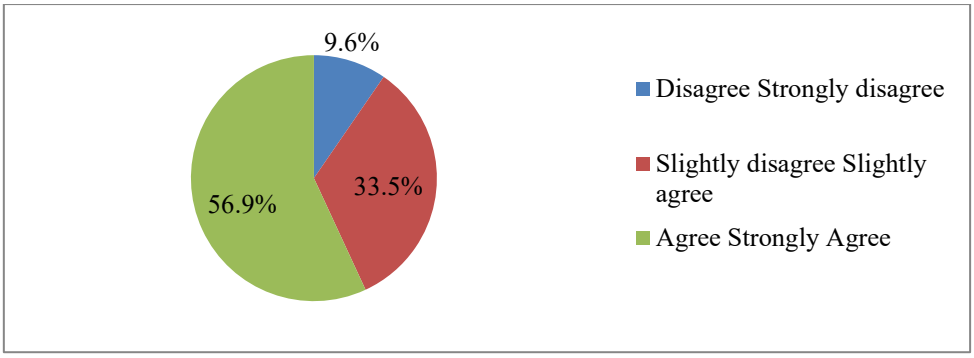


Figure 4. *Graphical representation of the distribution for the statement: In mathematics many things can be discovered and tried out by one self.*

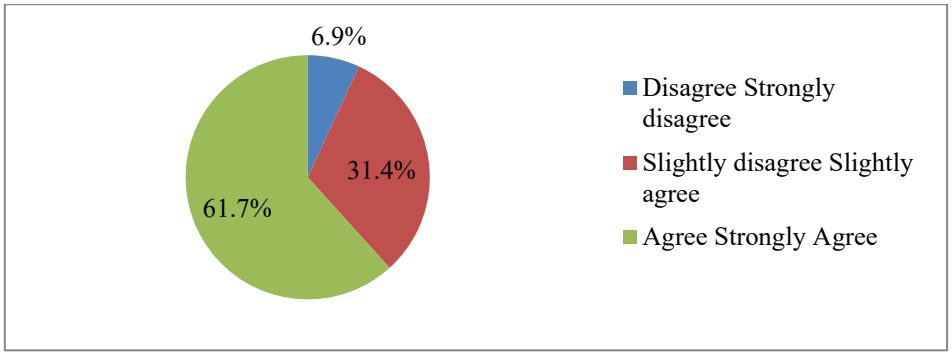


Figure 5. *Graphical representation of the distribution for the statement: Mathematical problems can be solved correctly in many ways.*

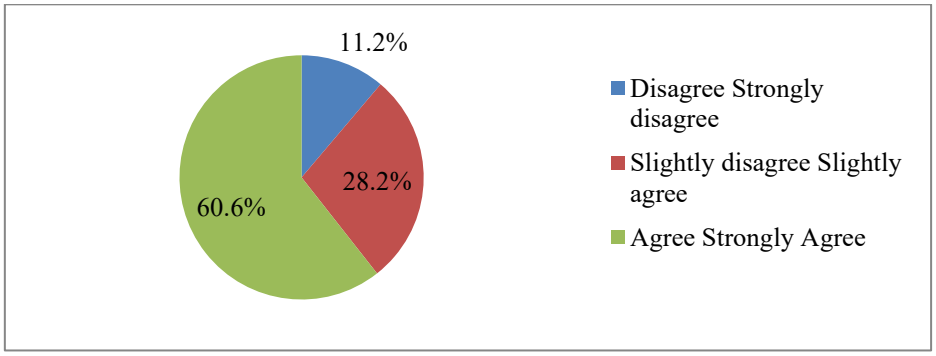


Figure 6. *Graphical representation of the distribution for the statement: Mathematics helps solve everyday problems and tasks.*

Presented below is a brief analysis of the database, focusing on findings related to the language of instruction (Macedonian and Albanian) for pupils in the subject of mathematics. Given that gender is a significant variable, all calculations will be performed separately for boys and girls. The t-test itself (Table 2) verifies the aforementioned difference as statistically insignificant (Sig.(2-tailed)=.950 for boys and Sig.(2-tailed)=.499 for girls is not less than .05), that is, pupils have similar opinions about *Mathematics as a Set of Rules and Procedures*, whereas for *Mathematics as a Process of Inquiry* the t-test verifies the aforementioned difference as statistically significant (Sig.(2-tailed)=.001 for boys and (Sig.(2-tailed)=.002 for girls is less than .05) i.e. pupils have different opinions about *Mathematics as a Process of Inquiry*.

Table 2. *Independent t-test: Beliefs about the NATURE OF MATHEMATICS*

	segment	*Language of instruction*	N	Mean	SD	Levene's test F	Levene's test Sig.	t	df	Sig.
male	Mathematics as a set of rules and procedures	Macedonian	54	2.72	.348	1.484	.226	0.063	97	.950
		Albanian	45	2.73	.313					
	Mathematics as a process of inquiry	Macedonian	54	2.68	.415	0.041	.840	**3.320****	97	**.001**
		Albanian	45	2.41	.371					
female	Mathematics as a set of rules and procedures	Macedonian	40	2.68	.329	0.663	.418	0.678	87	.499
		Albanian	49	2.63	.375					
	Mathematics as a process of inquiry	Macedonian	40	2.61	.354	2.533	.115	**3.271****	87	**.002**
		Albanian	49	2.31	.474					

Consequently, we will conduct a more in-depth analysis (verification) within the segment "Mathematics as a process of inquiry."

The subsequent independent t-test (Table 3) shows that the aforementioned differences between pupils who learn in Macedonian and those in Albanian language (within the segment "*Mathematics as a Process of Inquiry*") can be pinpointed more precisely to two (out of three) statements, for both genders: "Mathematical problems can be solved correctly in many ways" ($t=2.019$, $p<.05$ for males; $t=3.039$, $p<.01$ for females) and "Mathematics helps solve everyday problems and tasks" ($t=4.641$, $p<.01$ for male pupils; $t=3.682$, $p<.01$ for female pupils). As can be seen, no significant difference was observed regarding the statement: "In mathematics many things can be discovered and tried out by one self".

Table 3. *Independent t-test: segment of Mathematics as a Process of Inquiry*

	statements	*Language of instruction*	N	Mean	SD	Levene's test F	Levene's test Sig.	t	df	Sig.
male	In mathematics many things can be discovered and tried out by one self	Macedonian	54	2.52	.693	0.751	.388	.277	97	.782
		Albanian	45	2.56	.624					
	Mathematical problems can be solved correctly in many ways	Macedonian	54	2.76	.512	7.188	.009	**2.019***	97	**.047**
		Albanian	45	2.53	.588					
	Mathematics helps solve everyday problems and tasks	Macedonian	54	2.76	.512	8.240	.005	**4.641****	97	**.000**
		Albanian	45	2.16	.737					
female	In mathematics many things can be discovered and tried out by one self	Macedonian	40	2.40	.632	0.909	.343	.057	87	.955
		Albanian	49	2.41	.705					
	Mathematical problems can be	Macedonian	40	2.65	.533	4.607	.035	**3.039****	87	**.003**

solved correctly in many ways	Albanian	49	2.24	.723					
Mathematics helps solve everyday problems and tasks	Macedonian	49	2.78	.480	18.935	.000	**3.682****	87	**.000**
	Albanian	49	2.29	.764					

Discussions and conclusion

If pupils perceive mathematics as a set of rules and procedures, yet simultaneously as a field of exploration where they can arrive at different solutions to the same problem, this indicates a significant development in how they understand the nature of mathematics. Such an attitude reflects an integration of procedural and conceptual understanding, demonstrating that pupils are not content with the mere mechanical application of formulas, but instead seek the reasoning that lies behind them.

According to Grooteenberg and Marshman (2016), pupils aged 10–14, specifically those in grades 5 through 8 hold negative views regarding the nature of mathematics. They argue that pupils' beliefs about the nature of mathematics and how they understand the subject constitute a highly significant component of learning mathematics, often reflecting traditional views concerning the nature and application of the discipline.

Pupils who study in the Macedonian language and those who study in the Albanian language hold different views regarding the nature of mathematics. From the Table 2, we observe that our hypothesis H: There are statistically significant differences between groups of pupils regarding their beliefs about the nature of mathematics, is supported with regard to the aspect of "*Mathematics as a Process of Inquiry*"; specifically, pupils studying in the Albanian language hold different views compared to pupils studying in the Macedonian language (among both boys and girls) regarding the statements Mathematical problems can be solved correctly in many ways and Mathematics helps solve everyday problems and tasks.

Primary school children often do not perceive mathematics as a subject with applications in everyday life. Therefore, it is suggested that, in order to improve pupils' beliefs regarding the nature of mathematics, enhancements should be made to how mathematics is applied (Kelly, 2004).

Pupils who experience the study of mathematics as a process of inquiry tend to be more motivated, more proficient in critical thinking, and more open to alternative approaches to problem-solving. This serves as a clear indicator of the development indicator of high-level mathematical competencies, where the focus lies not only on the final result but also on the process by which that result is achieved.

The attitudes of pupils who study in the Albanian language regarding mathematics as a process of inquiry indicate that teachers need to make greater efforts to help pupils understand that mathematics is a subject with applications in everyday life. From a pedagogical perspective, this perception necessitates a shift in the teacher's role and in how the learning process is organized. The teacher should not merely be a transmitter of knowledge, but rather a guide and a catalyst for pupils' intellectual exploration. This entails creating problem-based scenarios, challenging questions, and tasks that stimulate independent thinking and logical reasoning. Through this process, rules and procedures are not learned in isolation; instead, they acquire meaning through application and continuous reflection.

In an era of rapid change, it is essential to redefine how we understand and learn mathematics. Implementing teaching practices that foster 21st-century competencies and connect to real life can help develop a deeper and richer understanding of mathematics.

Teachers should create more space for exploration and creativity in their instruction. Likewise, they should encourage individual discovery, embrace diverse solutions to the same problem, and increasingly link mathematics to real-life situations.

Although the beliefs expressed above may sound positive, they do not always translate into real progress especially if they are not accompanied by genuine commitment, motivation, and support. This gap between "what pupils say" and "what they demonstrate in practice" is significant for teachers, psychologists, and parents; it shows that merely expressing positive attitudes is not enough to guarantee success.

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