

CASIMIR EFFECT

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Abstract

If two parallel uncharged conductive plates placed close to each other in order micro-meters distance in a vacuum, it would appear an attractive interaction between these two plates. This interaction is theoretically explained by the difference between the vacuum energy outside the plates and the vacuum energy inside the plates. This effect is known as the "Casimir effect" by the name of theoretical physicist Hendrik Casimir, who first predicted the existence of this force. The Casimir's effect has been proved experimentally several times. In this paper, initially is given a historical context in which the theory of Casimir's effect was developed and a chronology of attempts to prove this effect experimentally. Then is calculated mathematically Casimir's force according to the procedure followed by Casimir. There is given also a short treatment of the impact of the geometric configuration on the Casimir's effect, and at the very bottom briefly are given several possible applications of the Casimir's effect on technology (especially in nanotechnology), theoretical physics, mathematics, etc.

Keywords: *Casimir's effect, Casimir's force, vacuum energy, zero-point energy, vacuum fluctuations, quantum electrodynamics.*

1. Introduction

In 1873 Van der Waals proved that neutral atoms can interact with each other even if the distribution of positive and negative electricity is completely symmetric, although in terms of classical electrodynamics there was no reason to interact. He justified this interaction with the probability that atoms can be transformed into electric dipole for a short time. Therefore, this is a convenient point to start with a brief history of Casimir's effect [1].

In an attempt to advance the Van der Waals theory, in 1948, Casimir published his paper in which he calculated the force between a neutral atom and an ideal conductivity plate. Later he also calculated the force between the two ideal conductivity plates.

The Casimir force is too small. To have an idea of what force it comes to, we can compare this force with the force that appears in some other well-known phenomena. For example, the force acting on a 1cm x 1 cm plates separated by 1 μ m apart is 0.13 μ N. This force is comparable to the Coulomb force between the nucleus and the electron in the Hydrogen atom, the gravitational force between two 0.5kg weights separated by 1cm. Or about 1/1000 the weight of a housefly [3]!

Due to the extremely small value, measuring this force requires too much precision in experimentation. Therefore, it has been very difficult to prove or reject Casimir's theory at the time when first appeared. Thus, in 1958, Sparnaay attempted to measure this effect, but the uncertainty of his instruments was 100%, and his result could not be considered as proof of the existence of the Casimir Effect [5].

In the 1990s two experiments were performed which provided proof of the existence of the Casimir Effect.

In 1996 Lamoreaux instead of two plates used a plate and a sphere to measure this effect.

So the sphere pulls the plate and because of the movement of the plate by the piezoelectric material this mechanical movement turns into an electric signal and by this indirectly was measured Casimir's force. The uncertainty of this experiment was 5% [5].

The second measurement was produced by Mohideen & Roy in 1998.

In 1998 Mohideen & Roy used a sphere and a cantilever beam, and in this case, it can change the angle of reflection of the laser light. In this experiment, the uncertainty was 1% [5]. Including this, it is considered that the effect of Casimir's is proved experimentally.

Scientists have been working for many years with the Casimir effect, only because of theoretical curiosity. But in recent years, interest in Casimir's effect has increased significantly to experimental physicists and engineers. That's because this effect can be used in the future for the construction of micro-machines, whose work requires extremely high precision [4].

2. Explanation of the Casimir Effect

In classical mechanics, the definition of vacuum was "what remained if you emptied a container of all its particles and lowered the temperature down to absolute zero"

However, in quantum mechanics such a definition for the vacuum is incorrect. This is because according to quantum mechanics, all fields, and in particular the electromagnetic field, have fluctuations. What in classical mechanics was called a vacuum, in fact, is not "empty" at all. Realistically, it is filled with virtual particles, which are in a continuous state of fluctuation. Virtual particle-antiparticle pairs can be created from a vacuum and annihilated back to vacuum. Photons (quanta of electromagnetic waves) are the dominant virtual particles in vacuum fluctuations but other particles produced as well. These virtual particles exist for a time dictated by Heisenberg Uncertainty relation:

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2} \quad (1)$$

So, in the time interval with the uncertainty Δt , the system energy will have the uncertainty ΔE . Therefore, at very small intervals Δt , in the universe, the law of energy conservation for ΔE can be broken. Since constant \hbar is too small, Δt and ΔE are also too small, however sufficient for the appearance of the particle-antiparticle pair.

Thus in quantum mechanics, since the vacuum cannot be defined as total emptiness, we can best present it as a superposition of different states of the electromagnetic field. When the photon appears and disappears, we say that the vacuum fluctuates.

However, vacuum fluctuations are not some abstraction of a physicist's mind. They have observable consequences that can be directly visualized in experiments on a microscopic scale.

Imagine trying to hold a pencil upright on the end of your finger. It will stay there if your hand is perfectly stable and nothing perturbs the equilibrium. But the slightest perturbation

will make the pencil fall into a more stable equilibrium position. Similarly, vacuum fluctuations cause an excited atom to fall into its ground state. This phenomenon is a consequence of vacuum fluctuations [4].

If the laws of classical electrodynamics were fully valid, the Casimir effect should not have existed. So, to explain this phenomenon we have to use quantum electrodynamics. More specifically, a phenomenon that is called "zero-point energy".

As we will see below, electromagnetic waves of all frequencies fluctuate in a vacuum. Zero-point energy represents the sum of the energies of all possible frequencies of the continual frequency spectrum. This means that zero-point energy is infinite!

But, if the vacuum energy is infinite, why we do not notice this energy? The reason why we don't notice it is that this energy cannot be measured since its present in the entire space. Therefore, what can be done under these circumstances is to measure eventual differences between energies at different points of the space. So if possible, let's change the vacuum conditions and measure the difference in energy of the changed and unchanged points.

As will be seen, if we set boundary conditions to a part of the space, these boundary conditions will cause differences in zero-point energies beyond these boundaries and within these boundaries.

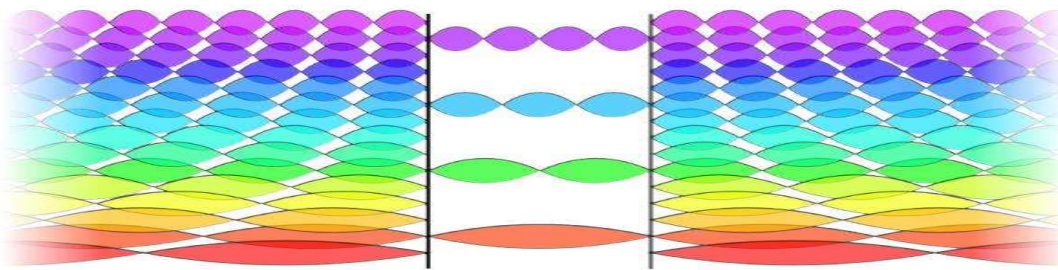


Figure 1. Outside the cavity formed by the plates, all vacuum frequencies are allowed. Within the cavity, however, the vacuum modes take on discrete frequencies. Changing the width of the cavity changes the density of modes relative to free space, which yields an energy difference [6]

Because of the boundary conditions, between plates cannot be fluctuations of the entire continuous frequency generation but only some of the discrete frequencies, while outside of the plates fluctuates the radiations of all frequencies, and this difference in energy translates into mechanical force.

In a sign of the Casimir force, it also affects the geometric shape of the structure. In some configuration forms, such as the flat capacitor configuration, this force is attractive and in some other forms, such as for instance in the spherical boundary configuration, Casimir's force is repulsive [6]. This feature makes Casimir's potentially suitable for use in nano-electro-mechanical devices.

3. Mathematical Calculation of the Casimir Force

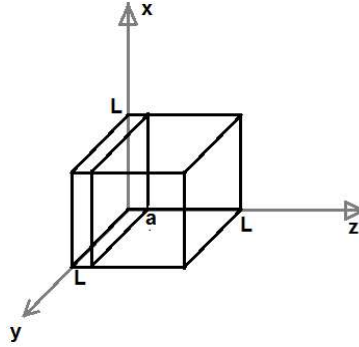


Figure 2. Cubic cavity with dimensions $L \times L \times L$ surrounded by walls with ideal conduction

Let it be a cubic cavity with dimensions $L \times L \times L$ surrounded by walls with ideal conduction. The interior of that cavity should be a vacuum. In the plane $z = a$ (so $a \ll L$) we put another plaque of ideal conductivity.

In a vacuum, angular frequency ω_k is related to the wave number k by the speed of light c :

$$\omega_k = ck \quad (2)$$

This allows us to express the zero-point energy of the field as a sum over quantized modes, identified by k_x , k_y and k_z , where:

$$k_x = \frac{n_x \pi}{L}, k_y = \frac{n_y \pi}{L}, k_z = \frac{n_z \pi}{L} \quad (3)$$

The electromagnetic field in the vacuum can be modeled as a quantum harmonic oscillator [5]. The harmonic oscillator Hamiltonian is:

$$H = \sum_k \hbar \omega_k \left(n + \frac{1}{2} \right) \quad (4)$$

The average energy of the oscillator in the ground state is:

$$\langle E \rangle = \langle 0 | H | 0 \rangle = \left\langle 0 \left| \sum_k \hbar \omega_k \left(n + \frac{1}{2} \right) \right| 0 \right\rangle = \frac{1}{2} \hbar \sum_k \omega_k = \frac{\hbar c}{2} \sum_k \sqrt{k_x^2 + k_y^2 + k_z^2} \quad (5)$$

Since L is too large, then we may regard k_x , k_y and k_z as continuous variables and $\Delta k_x =$

$$\frac{k_x}{n_x} = \frac{\pi}{L} \rightarrow 0,$$

$$\Delta k_y = \frac{k_y}{n_y} = \frac{\pi}{L} \rightarrow 0, \Delta k_z = \frac{k_z}{n_z} = \frac{\pi}{L} \rightarrow 0, \text{ so :}$$

$$\langle E \rangle = \frac{\hbar c}{2} \sum_{k_x} \sum_{k_y} \sum_{k_z} \sqrt{k_x^2 + k_y^2 + k_z^2} \Delta k_x \Delta k_y \Delta k_z \frac{L}{\pi} \frac{L}{\pi} \frac{L}{\pi} \approx 2 \cdot \frac{\hbar c}{2} \frac{L^3}{\pi^3} \iiint_0^\infty \sqrt{k_x^2 + k_y^2 + k_z^2} dk_x dk_y dk_z \quad (6)$$

Coefficient 2 is because the electromagnetic field energy is located in 2 polarizations.

The energy throughout the cub is:

$$\langle E_L \rangle = 2 \cdot \frac{\hbar c L^3}{2 \pi^3} \iiint_0^\infty \sqrt{k_x^2 + k_y^2 + k_z^2} dk_x dk_y dk_z \quad (7)$$

The energy in the part $z > a$ is (in this case $\frac{k_z}{n_z} = \frac{\pi}{(L-a)}$)

$$\langle E_{L-a} \rangle = 2 \cdot \frac{\hbar c L^2 (L-a)}{2 \pi^3} \iiint_0^\infty \sqrt{k_x^2 + k_y^2 + k_z^2} dk_x dk_y dk_z \quad (8)$$

In the case where $0 < z < a$ since a is too small, we can not consider k_z a continuous variable, so the triple sum returns to a double integral and single sum

$$\langle E_a \rangle = \frac{\hbar c L^2}{2 \pi^2} \sum_{k_z} \iint_0^\infty \sqrt{k_x^2 + k_y^2 + k_z^2} dk_x dk_y = \frac{\hbar c L^2}{2 \pi^2} \sum_{(0)1}^\infty \iint_0^\infty \sqrt{k_x^2 + k_y^2 + \frac{n_z^2 \pi^2}{a^2}} dk_x dk_y \quad (9)$$

Where the symbol (0) 1 at the lower limit of the sum means that when $n_z = 0$ the coefficient before the integral is 1 while when $n_z > 0$ then the coefficient before the integral is 2. This is because if $n_z > 0$ we have two present polarizations.

In the last expression, we move to polar coordinates and replace $n_z = n$ and we get:

$$\langle E_a \rangle = \frac{\hbar c L^2}{2 \pi^2} \sum_{(0)1}^\infty \frac{\pi}{2} \int_0^\infty \sqrt{r^2 + \frac{n^2 \pi^2}{a^2}} r dr \quad (10)$$

The difference in the energy of the restricted part and the rest of the cub is:

$$\delta E = E_a + E_{L-a} - E_L \quad (11)$$

$$\delta E = \frac{\hbar c L^2}{2 \pi^2} \left\{ \sum_{(0)1}^\infty \frac{\pi}{2} \int_0^\infty \sqrt{r^2 + \frac{n^2 \pi^2}{a^2}} r dr - \frac{a}{\pi} \int_0^\infty \left[\int_0^\infty \sqrt{r^2 + \frac{n^2 \pi^2}{a^2}} r dr \right] dk_z \right\} \quad (12)$$

These integrals are all divergent. However, knowing that high-frequency electromagnetic fields (such as gamma rays, etc.), the waves penetrate the plate without any obstruction (barrier), so functions under integral must be multiplied by a decreasing function of the form:

$$f\left(\frac{k}{k_m}\right) = \begin{cases} 1 & \text{for } k \ll k_m \\ 0 & \text{for } k \gg k_m \end{cases} \quad (13)$$

And making the substitution: $u = \frac{a^2 r^2}{\pi^2}$. We get:

$$\delta E = \frac{L^2 \hbar c \pi^2}{4 a^3} \left\{ \sum_{(0)1}^\infty \left[\int_0^\infty \sqrt{u + n^2} f(\pi \sqrt{u + n^2} / a k_m) du \right] - \int_0^\infty \left[\int_0^\infty \sqrt{u + n^2} f(\pi \sqrt{u + n^2} / a k_m) du \right] dn \right\} \quad (14)$$

Here we have the difference between a sum and integral of the same function. In such cases, the Euler-Maclaurin formula is used:

$$\sum_{(0)1}^{\infty} F(n) - \int_0^{\infty} F(n)dn = -\frac{1}{12}F'(0) + \frac{1}{24 \cdot 30}F'''(0) + \dots \quad (15)$$

We make the substitution $u + n^2 = w$ and for the function within the sum, respectively the integral, $F(n)$ we have:

$$F(n) = \int_{n^2}^{\infty} w^{1/2} f\left(\frac{w\pi}{ak_m}\right) dw \quad (16)$$

Using parital integration we get:

$$F(n) = \left| \begin{array}{l} u = f\left(\frac{w\pi}{ak_m}\right), \quad du = f'\left(\frac{w\pi}{ak_m}\right) \cdot \frac{\pi}{ak_m} dw \\ dv = w^{1/2} dw \quad v = \frac{2}{3} w^{3/2} \end{array} \right| = \frac{2}{3} w^{3/2} \cdot f\left(\frac{w\pi}{ak_m}\right) \Big|_{w=n^2}^{w=\infty} - \int_{n^2}^{\infty} \frac{2}{3} w^{3/2} f'\left(\frac{w\pi}{ak_m}\right) \cdot \frac{\pi}{ak_m} dw \quad (17)$$

The function f has been selected such that when $w \rightarrow \infty$, $f \rightarrow 0$. Such a function tends to zero faster than it tends to ∞ function $w^{3/2}$ (such as exponential function e^{-x} e.g.) therefore

$$\lim_{w \rightarrow \infty} \frac{2}{3} w^{3/2} \cdot f\left(\frac{w\pi}{ak_m}\right) = 0 \quad (18)$$

Also, function f tends to zero asymptotic so for large values of w its derivative is practically zero, so for values $w > n^2$ the integral is:

$$\int_{n^2}^{\infty} \frac{2}{3} w^{3/2} f'\left(\frac{w\pi}{ak_m}\right) \cdot \frac{\pi}{ak_m} dw = 0 \quad (19)$$

Therefore,

$$F(n) = -\frac{2}{3} n^3 \cdot f\left(\frac{n^2\pi}{ak_m}\right) \Big|_{w=n^2}^{w=\infty} \quad (20)$$

For derivatives of this function according to n , we have:

$$F'(n) = -2n^2 f\left(\frac{n^2\pi}{ak_m}\right) - \frac{2}{3} n^3 \cdot f'\left(\frac{n^2\pi}{ak_m}\right) \cdot \frac{\pi}{ak_m} = -2n^2 f\left(\frac{n^2\pi}{ak_m}\right)$$

$$F''(n) = -4n f\left(\frac{n^2\pi}{ak_m}\right)$$

$$F'''(n) = -4$$

Therefore,

$$F'(0) = 0$$

$$F'''(0) = -4$$

Thus, the difference in energy will be:

$$\begin{aligned} \delta E &= \frac{L^2 \hbar c \pi^2}{4a^3} \left\{ \sum_{(0)1}^{\infty} \left[\int_0^{\infty} \sqrt{u+n^2} f(\pi\sqrt{u+n^2}/ak_m) du \right] \right. \\ &\quad \left. - \int_0^{\infty} \left[\int_0^{\infty} \sqrt{u+n^2} f(\pi\sqrt{u+n^2}/ak_m) du \right] dn \right\} \\ &= \frac{L^2 \hbar c \pi^2}{4a^3} \left\{ -\frac{1}{12} \cdot 0 + \frac{1}{24 \cdot 30} \cdot (-4) + \dots \right\} \end{aligned}$$

$$\delta E = \frac{L^2 \hbar c \pi^2}{4a^3} \left(-\frac{1}{180} \right) \quad (21)$$

The energy (in unit area):

$$\frac{\delta E}{L^2} = -\frac{\hbar c \pi^2}{720 a^3} \quad (22)$$

So, for the Casimir force (in the unit of area) we have finally:

$$F = \frac{\hbar c \pi^2}{240 a^4} = \mathbf{0.13} \frac{1}{a^4} \frac{\mu\text{N}}{\text{cm}^2} \quad (23)$$

This was the mathematical expression of Casimir's force [1], the results of which were confirmed 50 years later.

4. The role of geometry in the effect of Casimir

Based on the explanation given for Casimir's force appearing in the case of two parallel plates, there is no reason for this force not to appear in other geometric configurations.

One of the possible configurations is the spherical configuration. One of the simplest possible configurations is that of a conducting spherical shell. Given the attraction between the parallel plates and atoms discussed in the previous sections, one might expect that the Casimir force would pull a spherical shell inward, as vacuum fluctuations outside the cavity overwhelmed those within.

Casimir suggested in 1956 that this might provide a solution to a longstanding problem in Physics: the electron radius. To avoid an infinite charge density (and self-energy) for a point electron, some physicists suggested that the electron's charge be spread out in a spherical configuration. However, such a charge distribution would exert an outward electrostatic force which would cause the electron shell to expand. Poincaré suggested the introduction of ad-hoc forces to ensure the electron's stability. Casimir proposed that these forces (by analogy with his parallel plate's derivation) could be accounted for by the zero-point energy of the configuration. Unfortunately, this turned out not to be the case [6].

Inspired by this Casimir's idea, Timothy Boyer in [8] had mathematized the Casimir's force for spherical configuration. The mathematical apparatus used by Boyer is, in essence the same as that used by Casimir, when boundary conditions are imposed on the vacuum at a radial sphere boundary and the rest of the universe with a radius of $R \rightarrow \infty$.

The final expression that Boyer came up with was quite complicated, but using numerical analysis, he gained the approximate expression for vacuum energy, spherical configuration, in the form:

$$\langle E \rangle \approx +\mathbf{0.09} \frac{\hbar c}{2a} \quad (24)$$

Boyer's results were reconfirmed by Davies, Balian, and Duplantier, Leseduarte and Romeo [9].

What's interesting to note in this result is that the vacuum energy inside the sphere is positive! This means that a conductive sphere in a vacuum tends to expand due to the Casimir effect. Because of this, Casimir's force could not be the force that holds the "spherical" electron stable.

There are several attempts to explain this result, and none are definitive. One possible explanation is that this repulsive force comes as a result of the dipole interaction formed to cancel the electric field at the border. Another explanation is that within the sphere the density of the states is higher than outside the sphere (according to Boyer's result the stable waves within the sphere are described by Bessel's functions, which allows a greater density than the usual sinusoids) therefore the force is expanding etc.

From here, it is seen that the configuration geometry has a crucial role in the nature and value of Casimir force.

5. Experimental proof of Casimir's effect

In 1958, Sparnaay attempted for the first time to measure Casimir's force between two reflective plates. Although the uncertainty of this measurement was approximately 100%, and in experimental terms, it could be said that the experiment had failed, this experiment was important in the sense that it was the first step towards proving the effect of Casimir and showed what needs to be improved in future versions of the experiment [10].

Because of the high precision requirements that exceeded the technical possibilities of the time, this experiment was no longer tried until 1997 [11]. During this time Sabinsky and Anderson (in 1972) managed to demonstrate some of the effects of vacuum energy [15].

While in 1997 Lamoreaux demonstrated the Casimir effect using the plate-sphere technique, which has now become a standard technique in experiments related to the Casimir effect [11].

Of course, in theory, the interaction between two flat plates is a simple problem to solve, but this is not true even in the experimental aspect. In this kind of experiment, the distance between the plates, which is nanometer order, should be held constant, also the plates should be almost completely parallel. So the angle between the two plates should ideally be zero with tolerance to 10^{-5} rad. It is very difficult to achieve such precise experiments; therefore, the Casimir effect is not used plate-plate configuration but sphere-plate. This is possible thanks to the Proximity Force Theorem, or PFT, which models the sphere in terms of small plane segments with corrected areas [7]. By dividing the sphere into small flat parts, theoretically can be calculated that the magnitude of Casimir's force between a sphere and a plaque is:

$$F(a) = 2\pi R \frac{1}{3} \frac{\pi^2}{240} \frac{\hbar c}{a^3}$$

where R is the radius of the sphere and a is the distance between the spherical surface and the plate [6].

The surfaces used in the experiment were small lenses, with a diameter of 2.54 cm, covered with a copper vapor layer of 0.5 μm and a gold layer also 0.5 μm . The sphere was associated with a micro-positioning apparatus which allow controlling the distance between the sphere and the plate, with nanoscale precision, through a series of a piezo-transducer¹. The Plate, on the other hand, was located in a small "rod", which enables rotation of the plate for small angles, depending on the force between it and the sphere.

A plate, on the other hand, was located in a small "rod", which enables rotation of the plate for small angles, depending on the force between it and the sphere.

Despite numerous complications, this experiment was able to prove Casimir's force, with an uncertainty of up to 5%, for distances smaller than $0.6\text{ }\mu\text{m}$. This uncertainty, although it represents a tremendous improvement compared to Sparnaay's experiment, could still be considered high enough to consider the correction of Casimir's force due to thermal effects.

Subsequent experiments have used more and more perfect techniques to achieve uncertainty even less than 1% [12].

After the success of Lamoreaux, many other researchers tried to refine methods for measuring the Casimir effect.

Umar Mohideen together with his co-workers at the University of California in 1998, attached a polystyrene sphere with a diameter of $200\text{ }\mu\text{m}$ to the tip of atomic force microscope (Figure 3). In a series of experiments, they brought the sphere that was coated with aluminum or gold, near a flat disk (coated with the same metal) at a distance of $0.1\text{ }\mu\text{m}$. The resulting force was measured by the deviation of the laser beam. With this technique, these researchers were able to measure the Casimir force to within 1% of the expected theoretical value.

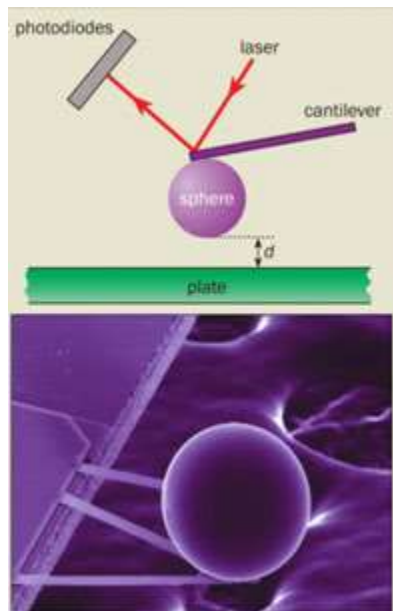


Figure 3. Atomic force microscopy

Also, Thomas Ederth at the Royal Institute of Technology in Stockholm has used an atomic force microscope to study the Casimir effect. He measured the force between two gold-coated cylinders that were arranged at 90° to each other and that were as little as 20 nm apart. His results agreed to within 1% of theory (Figure 4).

In Figure 4 is given the scheme of the Ederth experiment. The upper cylinder can be lowered using the piezoelectric tube, which changes shape when a voltage is applied. The

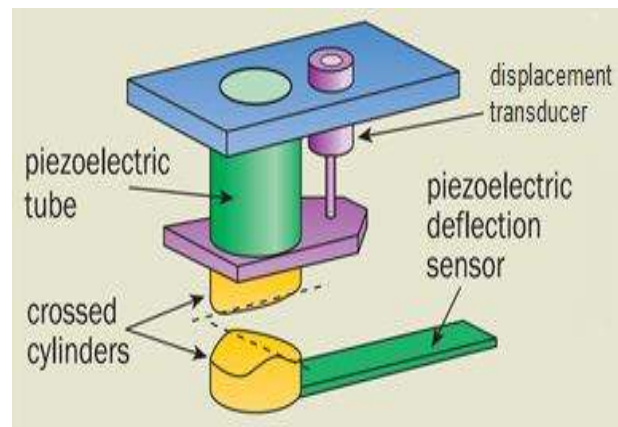


Figure 4. Measuring the force between two gold-coated cylinders

lower cylinder is mounted on a piezoelectric deflection sensor (known as a bimorph spring) that generates a charge when it is bent. When the two cylinders are close together, the Casimir force causes the lower cylinder to be attracted to the upper one, thereby deflecting the spring in the process. The linearly variable displacement transducer (LVDT) monitors the nonlinear expansion of the piezotube [14].

An attempt to experimentally measure the Casimir force, the initial configuration plaque-plaque was made in 2002 by Gianni Carugno, Roberto Onofrio along with his co-workers at the University of Padova in Italy. They measured the force between a plate coated with chrome and a flat surface of a microscopic club composed of the same material. These two bodies were removed from each other for 0.5-3 μm [16]. The result of these researchers agrees with the theoretical result of the Casimir force for this type of configuration within the 15% error. This poor match between experimental and theoretical results indicates the extremely complex difficulties associated with this type of experiment [14].

Experiments have also been made in the case where there is a dielectric between the plates instead of the vacuum, i.e. measurement of the Casimir force changes at a certain distance when changing the dielectric properties of the medium between the two plates.

An appropriate practical way to accomplish such an experiment is to use Hydrogen Switchable Mirrors - HSM.

Some metals such as Mg_2Ni alternate between being reflective when in the air and being transparent when appearing in hydrogen. If such material placed between the plates, the diversion of their reflective properties affects the allowable frequencies between the plates and thus causing a slight change in the Casimir force.

In 2004, Iannuzzi, Lisanti, and Capazzo tried to measure these changes in an experiment using a sphere and a plaque at 70-400 nm distances. The plate was gold and spheres were coated with a material HSM [17].

Force measurements were made by a piezoelectric transducer connected to the gold plate. Although transitions HSM (from reflective properties in transparent properties) were successfully implemented, in both cases, they measured the Casimir forces were exactly the same!

Further, in 2006 de Man and Iannuzzi predicted theoretically the conditions that had to be fulfilled to notice the changes in Casimir's force. [18] Since the dielectric properties of HSM materials are known only for waves of 0.2 to 2.5 μm length we cannot know the dielectric behavior of HSM at other lengths of value. It is possible that the mirror surface is not transparent at higher frequencies. If this is true, this would explain why force differences in the case of HSM alternatives are smaller than predicted.

De Man and Iannuzzi concluded that the decline in the Casimir's force should be dependent behavior (imagined) of the dielectric at different frequencies. Because of this variability, they concluded that the force-measured difference was within the limits of the uncertainty of their experiment and this could be detected by further perfecting the experimentation techniques. This dependence has not yet been proved experimentally. Further experiments on the dependence of the Casimir force of HSM effects can bring significant results not only for a deeper recognition of the Casimir force but also in the implementation of these

effects in the science of materials, an issue which brings us to the next section of this paper: The Casimir effect applications.

6. Application of the Casimir effect

The simplest application of Casimir's effect is the mechanical analogy of this effect in determining the attractive force between two ships in a rough sea owing to modification of the wave structure in the region between the ships [19].

In nanotechnology, Casimir's force dominates over other forces operating between the components of different systems. Clearly, Casimir's force will have an important role in nanotechnology. But for the moment it is difficult to predict whether it will have a beneficial role in nanotechnology or it will be an obstacle that needs to be avoided. This is because the force is very non-linear. This kind of force cannot support steady mechanical oscillators. So for the moment, the Nanoengineering uses remain speculative [20].

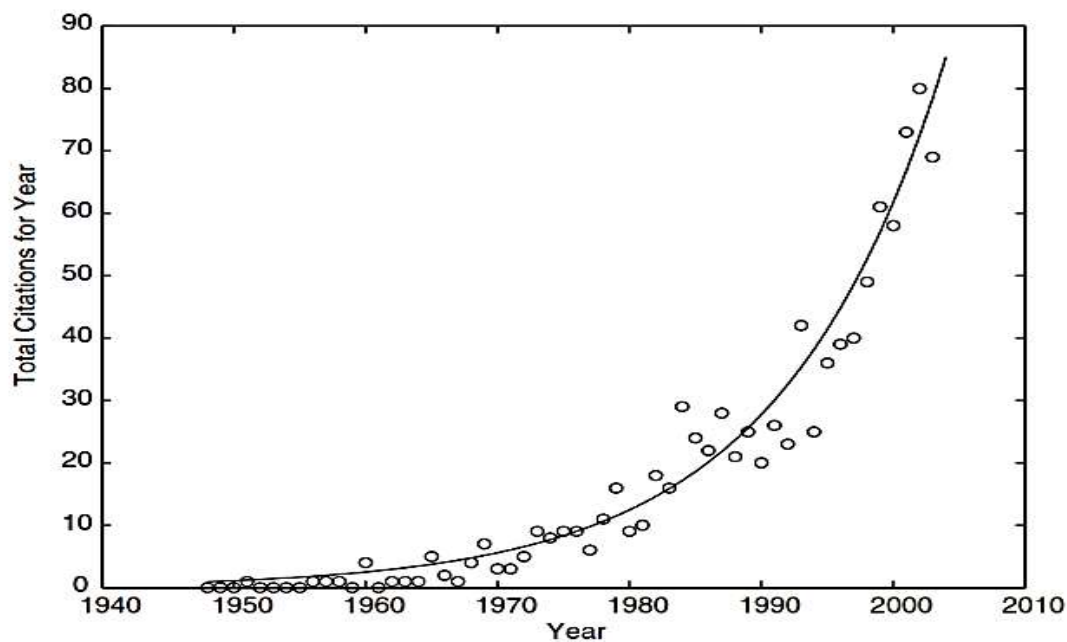


Figure 5. Casimir effect citations

Since the Casimir effect belongs to the phenomenon of the nanometric order, it is expected to be extremely applied in the future. This is also demonstrated by the exponential increase in the number of citations of the Casimir effect per year, as seen in fig.5.

In cosmological problems, the Casimir effect can also be related. The vacuum polarization resulting from the Casimir effect can drive the inflation process in the early stages of the universe. [4]

Casimir force fundamentally influences the performance and yield of nanodevices. Most present-day nanomechanical devices are based on thin cantilever beams above a silicon substrate fabricated by photolithography followed by dry and wet chemical etching. The cantilever's motion is greatly influenced by the Casimir force which dominates over other forces at a distance of a few nanometers. Thus, movable components in nanoscale devices

fabricated at distances less than 100nm between each other often stick together due to the strong Casimir force, leading to the collapse of a movable element to the substrate or the collapse of neighboring components during nanoscale device operation. Therefore, this phenomenon severely restricts the yield and operation of the devices, and the Casimir forces might well set fundamental limits on the performance and the possible density of devices that can be optimized on a single chip [4].

In 1941, Schiff suggested that the formation of films of superfluid helium on the walls of containers is due to the van der Waals attraction between the substrate and the helium atoms. Measurements of the thickness of liquid helium films are in good agreement with the generalized theory. More recent studies have shown that liquid helium will not wet a caesium film and this has been experimentally demonstrated. This discovery has a practical application as an important technique in low-temperature physics because an evaporated Cs ring interrupts superfluid film flow and eliminates the associated heat load. In a series of remarkable experiments, it was demonstrated that liquid water does not wet the surface of the ice. This theory has also been applied to the wetting of water on indium-tin-oxide films on windshields and is but one of the far-reaching applications of the Casimir force. [20]

In physical mathematics, the study of the Casimir effect has stimulated recent developments in divergence series renormalization techniques, based on zeta functions etc. [4].

7. Conclusions

The electrodynamic oscillator (or electromagnetic fluctuations or virtual photons) in the ground it states in vacuum swings at any frequency. Its energy is infinite!

The electrodynamics oscillator in the ground state within the parallel vacuum plates, due to the boundary conditions, swings only at certain discrete frequencies.

Between these parallel plates appear attractive forces that are proportional to a^{-4} . The presentation of the force is known as the Casimir effect. This effect has been experimentally proven more than once.

We can say that if the laws of classical electrodynamics were valid, this effect would not be presented. The presentation of this effect is one more argument for the validity of quantum electrodynamics.

Geometric Configuration affects the value and the sign of Casimir's force. For example, when the configuration is spherical Casimir's force is expanding.

The preliminary conclusion rejects the idea of Casimir that is exactly the Casimir force that maintains a stable electron as a sphere with a certain radius.

The Casimir force is the most important force in Nanotechnological devices therefore Casimir effect is expected to play an important role in nanotechnology. This role for now, cannot be clearly defined.

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